Some misc practice problems for exam #5

① LU of singular matrices

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \rightarrow \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{L} \cdot \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{U}$$

$$\begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \longrightarrow \underbrace{\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}}_{P} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}_{L} \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}}_{U}$$

② Condition number for permutation matrices

Ans. Permutation matrices are orthogonal matrices (i.e $P^{-1} = P^T$; $\det(P) = 1$).
Such matrices have a condition number of 1.

③ Determinant of A using LU?

Ans.

$$PA = LU$$
$$A = P^T LU$$
$$\det(A) = \det(P^T) \cdot \det(L) \cdot \det(U)$$
$$\det(A) = \det(L) \cdot \det(U)$$

① 3^{rd} derivative of interpolant

Ans.

$$V\alpha = f \quad - ⓪$$
$$\alpha = V^{-1} f$$

Diff the basis: ⓪
$$V'\alpha = \hat{f}' \quad - ①$$

subs in ①,
$$V'V^{-1} f = \hat{f}' \quad - ②$$

Writing with original basis and different coeffs, we can write
$$V\alpha' = \hat{f}' \quad - ③$$
$$\alpha' = V^{-1} \hat{f}'$$

from ②, $\alpha' = V^{-1} V' V^{-1} f$

Diff the basis ③

$V'\alpha' = \hat{f}''$ — ④

Subs in ④,

$V' V^{-1} V' V^{-1} f = \hat{f}''$

Repeating the steps above:

$V \alpha'' = \hat{f}''$

$\alpha'' = V^{-1} \hat{f}''$

$\alpha'' = V^{-1} V' V^{-1} V' V^{-1} f$

Subs back in ($V'\alpha'' = \hat{f}'''$):

$V' V^{-1} V' V^{-1} V' V^{-1} f = \hat{f}'''$