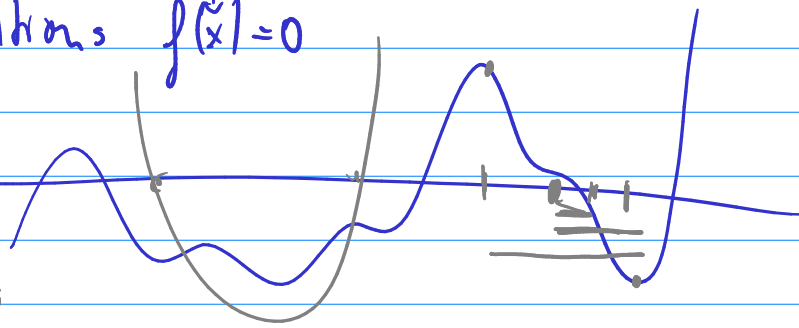
Overview

- Convergence rate

  linear: $e_{k+1} \leq C \cdot e_k$

  quadratic: $e_{k+1} \leq C \cdot e_k^2$

- Solve nonlinear equations $f(\vec{x}) = 0$



Bisection: linearly convergent

Plan

- solution method that's faster than linear: Newton's method

  (adv. / disadv. / fix)

- Solve nonlinear equations in n dim.
- Optimization | in 1D $f(x)=0$ ⇝ $f(x)$ as small as possible
  | in nD

Solving Nonlinear Equations

- What is the goal here?

Bisection Method

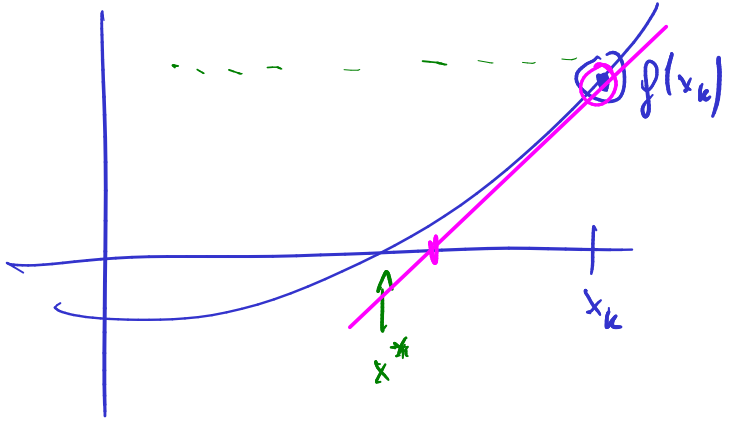
Demo: Bisection Method

- What's the rate of convergence? What's the constant?

Newton's Method

- Derive Newton's method.

$$f(x_k + s) \approx f(x_k) + f'(x_k) \cdot s = \tilde{f}(x_{k+s})$$



Idea: Instead of $f(x_k+s)=0$

Solve $\tilde{f}(x_{k+s})=0$

$$f(x_k) + f'(x_k) \cdot s = 0$$

$$s = -\frac{f(x_k)}{f'(x_k)}$$

Newton's method $\longrightarrow$ $x_{k+1} = x_k + s = x_k - \frac{f(x_k)}{f'(x_k)}$

Demo: Newton's method
Demo: Convergence of Newton's Method

- What are some **drawbacks** of Newton?

Secant Method

- What would Newton without the use of the derivative look like?

Idea behind Newton: $x_{k+1} = x_k - \frac{f(x_k)}{\text{slope (at } x_k)}$

↳ slope at x_k: $f'(x_k)$

↳ approximate slope at x_k:

$$f'(x_k) \approx \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

rate of conv.: 1.6

Demo: Secant Method
In-class activity: Nonlinear equations in 1D

Potential fix for problems w/ Newton/secant if slope is close to 0: limit step size

"trust region method"

16 Solving Many Equations

Solving Nonlinear Equations

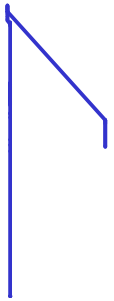
- What is the goal here?

$f(x) = 0$

$f(x, y) = 0$

$g(x, y) = 0$

$x = x^2 + y^2$



$x^2 - 3 \qquad x_0 = 1$

$f(1) + (x-1) \cdot f'(1)$

Newton's method

- What does Newton's method look like in n dimensions?

Newton: Example

- Set up Newton's method to find a root of

$$f(x, y) = \begin{pmatrix} x + 2y - 2 \\ x^2 + 4y^2 - 4 \end{pmatrix}.$$

Demo: Newton's method in n dimensions