Numerical Methods

CS 357 - Spring 2017

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Introduction

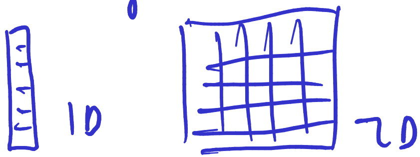
Numerical Methods: What?

- ‘Numerical’?
- ‘Method’?

Numerical

$\pi = 3.14159\ldots$

- Number representation
- n-dimensional arrays



Method?

- Math idea
- Possibly many algorithms
  - ↳ differ in runtime
  - ↳ accuracy
  - ↳ efficiency

Method = Math
+ Algorithm
+ Complexity / Efficiency
+ Accuracy

Accuracy

- Why might a numerical method **not give the right answer?** (i.e. be inaccurate)

Computers/resources are finite

→ accurate answer might not be feasible

→ error amplification ("conditioning")

Demo: Waiting for 1

Numerical Experiments

Model:

- Small-scale behavior easy to describe
- Large-scale behavior desired, but hard to understand

Demo: Brownian Motion

Numerical Experiments

- What are we going to want to know about a numerical experiment?
  - What question are we trying to answer?
  - What answer did the experiment provide?
    - ↳ How confident are we in that answer?
  - How expensive was the simulation?
  - How does the expense vary?
  - Reproducible? Repeatable?
  - Efficient?

Class web page

`bit.ly/cs357-s17`

- Assignments
  - HW0!
  - Pre-lecture quizzes
  - In-lecture interactive content (bring computer or phone if possible)
- Exams
- Class outline (with links to notes/demos/activities/quizzes)
- Scribbles
- Virtual Machine Image
- Piazza

- Policies
- Video
- Interactive Questions
- Calendar
    - Office Hours

In-class activity: Complexity of Matrix-Matrix Multiplication

$$\text{Time}(n) \approx c \cdot n^3$$

$$\text{Time}(2n) \approx c \cdot (2n)^3$$

$$= 2^3 \cdot c \cdot n^3 = 8 \cdot \text{Time}(n)$$

Recap: Understanding Asymptotic Behavior, $O(\cdot)$ Notation

Demo: Cost of Matrix-Matrix Multiplication

- Can we say anything exact about our results?
- How do we say something exact without having to predict individual values exactly?

Making Predictions with $O(\cdot)$-Notation

- Suppose you know that $\text{Time}(n) = O(n^2)$. And you know that for $n_1 = 1000$, the time taken was 5 seconds. Estimate how much time would be taken for $n_2 = 2000$.