- CBTF
- Quiz 3 ⟵ due noon Fri.
- HW0 ⟵
- HW1 ⟵ due Friday
- Office Hour Changes

Overview

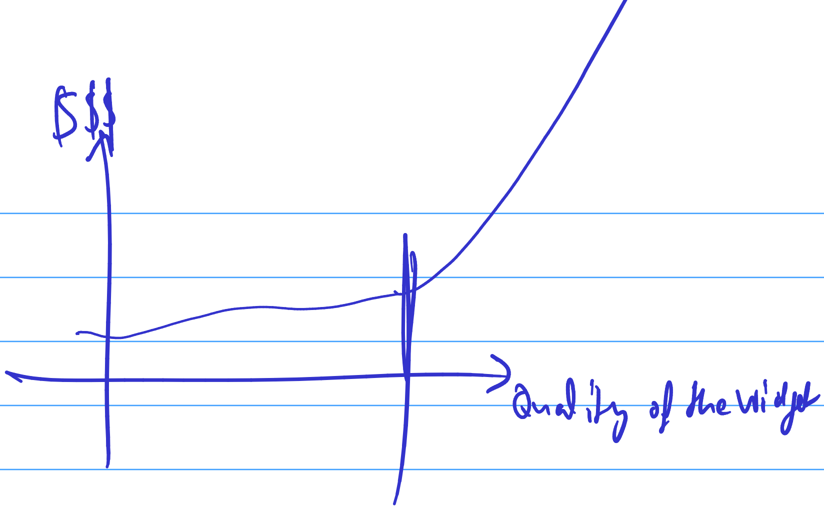
- numpy
- polynomials
  - ↳ Taylor

Programming Language: Python/numpy

- Reasonably readable
- Reasonably beginner-friendly
- Mainstream (top 5 in 'TIOBE Index')
- Free, open-source
- Great tools and libraries (not just) for scientific computing
- Python 2/3? 3!
- `numpy`: Provides an array datatype
  Will use this and `matplotlib` all the time.
- See class web page for learning materials

- **Demo:** Python
- **Demo:** numpy
- **In-class activity:** Image Processing

Outline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Quality of the widget

Why polynomials?

$$a_3x^3 + a_2x^2 + a_1x + a_0$$

- How do we write a general degree n polynomial?

$$\sum_{i=0}^{n} a_i x^i = a_0 \cdot \underbrace{x^0}_{1} + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3 \cdots$$

- *Why* polynomials and not something else?

  - easy to construct
  - easy to eval

Reconstructing a Function From Derivatives

- If we know $f(x_0)$, $f'(x_0)$, $f''(x_0)$, can we approximately reconstruct the function as a polynomial p?

$$p(x) = ??? + ???x + ???x^2 + \cdots$$

Reconstructing a Function From Derivatives

$p''(0) = f''(0)$ yields $c = f''(0)/2$.

$$f'''(x) = 3 \cdot 2d + 4 \cdot 3 \cdot 2ex$$

$p'''(0) = f'''(0)$ yields $d = f'''(0)/3!$. (and so on)

Found: *Taylor series approximation*.

$$f(0 + x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \cdots$$

General pattern:

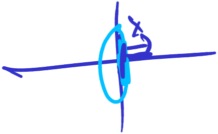
$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!}x^i$$

Demo: Polynomial Approximation with Derivatives (Part I)

Shifting the Expansion Center

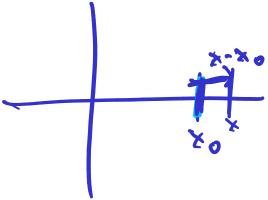
- Can you do this at points other than the origin?

$$f(0+x) \approx f(0) + f'(0)\cdot x + f''(0)\cdot \frac{x^2}{2}$$



$0 \mapsto x_0$

$x \mapsto x - x_0$



$$f(x) = f(x_0 + (x - x_0))$$
$$= f(x_0) + f'(x_0)\cdot(x - x_0) + f''(x_0)\cdot\frac{(x-x_0)^2}{2}$$

Errors in Taylor Approximation (I)

- Can't sum infinitely many terms. Have to *truncate*. How big of an error does this cause?

  Demo: Polynomial Approximation with Derivatives (Part II)