

- CDTF
- Quiz 3 ← due noon Sat.
- HW0 ←
- HW1 ← due Friday
- Office Hour Changes

Overview

- numpy
- polynomials
↳ Taylor

Programming Language: Python/numpy

- ▶ Reasonably readable
- ▶ Reasonably beginner-friendly
- ▶ Mainstream (top 5 in 'TIOBE Index')
- ▶ Free, open-source
- ▶ Great tools and libraries (not just) for scientific computing
- ▶ Python 2/3? 3!
- ▶ `numpy`: Provides an array datatype
Will use this and `matplotlib` all the time.
- ▶ See class web page for learning materials

- ▶ **Demo:** Python
- ▶ **Demo:** numpy
- ▶ **In-class activity:** Image Processing

Outline

Python, Numpy, and Matplotlib

Making Models with Polynomials

Making Models with Monte Carlo

Error, Accuracy and Convergence

Floating Point

Modeling the World with Arrays

- The World in a Vector

- What can Matrices Do?

- Graphs

- Sparsity

Norms and Errors

The 'Undo' Button for Linear

Operations: LU

LU: Applications

- Linear Algebra Applications

- Interpolation

Repeating Linear Operations:

Eigenvalues and Steady States

Eigenvalues: Applications

Approximate Undo: SVD and Least

Squares

SVD: Applications

- Solving Funny-Shaped Linear

- Systems

- Data Fitting

- Norms and Condition Numbers

- Low-Rank Approximation

Iteration and Convergence

Solving One Equation

Solving Many Equations

Finding the Best: Optimization in 1D

Optimization in n Dimensions



Why polynomials?

$$\underline{a_3}x^3 + \underline{a_2}x^2 + \underline{a_1}x + \underline{a_0}$$

- How do we write a general degree n polynomial?

$$\sum_{i=0}^n a_i x^i = a_0 \cdot \frac{x^0}{1} + a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3 \dots$$

- Why polynomials and not something else?

- easy to construct
- easy to eval

Reconstructing a Function From Derivatives

- If we know $f(x_0)$, $f'(x_0)$, $f''(x_0)$, can we approximately reconstruct the function as a polynomial p ?

$$p(x) = \underbrace{???}_a + \underbrace{???}_b x + \underbrace{???}_c x^2 + \dots$$

$f(0)$ $f'(0)$ $f''(0)/2$ $f'''(0)/6$

$$\underline{f(0) = p(0) = a} \quad \Rightarrow \quad a = f(0)$$

$$p'(x) = b + \underline{2cx + 3dx^2}$$

$$\underline{f'(0) = p'(0) = b}$$

$$p''(x) = 2c + \underline{6x}$$

$$f''(0) = p''(0) = 2c$$

Reconstructing a Function From Derivatives

$p''(0) = f''(0)$ yields $c = f''(0)/2$.

$$f'''(x) = 3 \cdot 2d + 4 \cdot 3 \cdot 2ex$$

$p'''(0) = f'''(0)$ yields $d = f'''(0)/3!$. (and so on)

Found: *Taylor series approximation*.

$$f(0 + x) \approx f(0) + f'(0)x + \frac{f''(0)}{2}x^2 + \dots$$

General pattern:

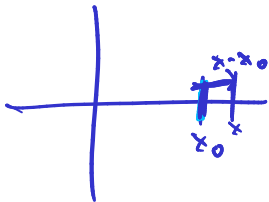
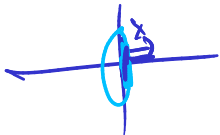
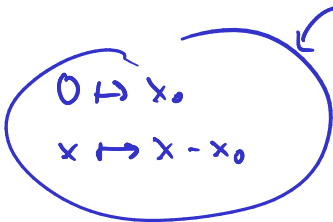
$$f(x) = \sum_{i=0}^{\infty} \frac{f^{(i)}(0)}{i!} x^i$$

Demo: Polynomial Approximation with Derivatives (Part I)

Shifting the Expansion Center

- Can you do this at points other than the origin?

$$f(0+x) \approx f(0) + f'(0) \cdot x + f''(0) \cdot \frac{x^2}{2}$$



$$\begin{aligned} f(x) &= f(x_0 + (x - x_0)) \\ &= f(x_0) + f'(x_0) \cdot (x - x_0) \\ &\quad + f''(x_0) \cdot \frac{(x - x_0)^2}{2} \end{aligned}$$

Errors in Taylor Approximation (I)

- Can't sum infinitely many terms. Have to *truncate*. How big of an error does this cause?

Demo: Polynomial Approximation with Derivatives (Part II)