Overview

Computational lin alg-

Announcements

Examlet 2 ✓

Office hour changes ✓

HW4 ✓

IEF ✓

Outline

Some Perspective

- We have so far (mostly) looked at what we can do with single numbers (and functions that return single numbers).
- Things can get *much* more interesting once we allow not just one, but *many* numbers together.
- It is natural to view an *array of numbers* as one object with its own rules.

  The simplest such set of rules is that of a vector.
- A 2D array of numbers can also be looked at as a matrix.
- So it's natural to use the tools of computational linear algebra.
- 'Vector' and 'matrix' are just *representations* that come to life in many (*many!*) applications. The purpose of this section is to explore some of those applications.

Vectors

What's a vector?

$\begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$

a vector is just a thing $\in$ Vector Space V

$3x + 5x = 8x$

+ "arithmetic makes sense"

$\vec{x} \in V \quad y \in V$

$\vec{x} + \vec{y} \in V$

$\alpha \vec{x} \in V \quad \alpha \in \mathbb{R}$

Vectors from a CS Perspective

What would the concept of a vector look like in a programming language (e.g. Java)?

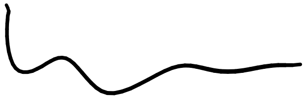
```
interface Vector {
    Vector add( Vector other);
    Vector scale( float scalar);
}
```

Vectors in the 'Real World'

Demo: Images as Vectors

Demo: Sound as Vectors

Demo: Shapes as Vectors



Outline

Matrices

What does a matrix do?

It represents a *linear function* between two vector spaces $f : U \to V$ in terms of bases $\boldsymbol{u}_1, \ldots, \boldsymbol{u}_n$ of U and $\boldsymbol{v}_1, \ldots, \boldsymbol{v}_m$ of V. Let

$$\boldsymbol{u} = \alpha_1 \boldsymbol{u}_1 + \cdots + \alpha_n \boldsymbol{u}_n$$

and

$$\boldsymbol{v} = \beta_1 \boldsymbol{v}_1 + \cdots + \beta_m \boldsymbol{v}_m.$$

Then f can *always* be represented as a matrix that obtains the βs from the αs:

$$\begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \vdots \\ \beta_m \end{pmatrix}.$$

Example: The ‘Frequency Shift’ Matrix

Assume both $\boldsymbol{u}$ and $\mathbf{v}$ are linear combination of sounds of different frequencies:

$$\boldsymbol{u} = \alpha_1 \boldsymbol{u}_{110\text{ Hz}} + \alpha_2 \boldsymbol{u}_{220\text{ Hz}} + \cdots + \alpha_4 \boldsymbol{u}_{880\text{ Hz}}$$

(analogously for $\boldsymbol{v}$, but with βs). What matrix realizes a ‘frequency doubling’ of a signal represented this way?

$$\begin{array}{cccc} \alpha_1 & \alpha_2 & \alpha_3 & \alpha_4 \end{array}$$

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \end{pmatrix} = \begin{pmatrix} 0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix}$$

$$\text{output} = 0\, u_{110} + \alpha_1 u_{220} + \alpha_2 u_{440} + \alpha_3 u_{880}$$

Matrices in the 'Real World'

What are some examples of matrices in applications?

Demo: Matrices for Geometry Transformation
Demo: Matrices for Image Blurring
In-class activity: Computational Linear Algebra

Outline

Graphs as Matrices

How could this (directed) graph be written as a matrix?