Overview

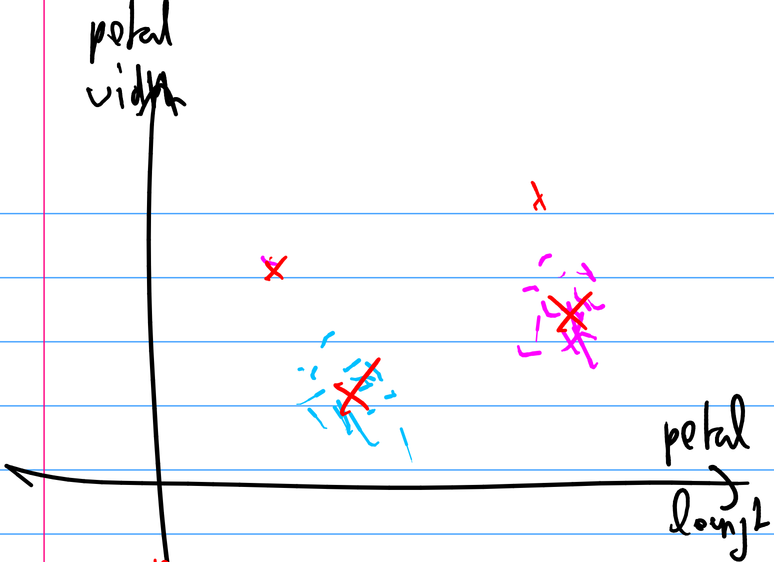
- LU

Examlet 4

(∂)

Fri / Sa / Sn

700



$$\frac{1}{n}\sum_{i=1}^{n} \vec{x}_i$$

General LU Factorization (Gaussian Elimination)

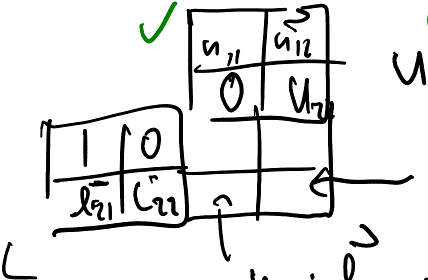
$$A = \begin{bmatrix} a_{11} & \mathbf{a_{12}} \\ \mathbf{a_{21}} & A_{22} \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 \\ \mathbf{l_{21}} & L_{22} \end{bmatrix}}_{L} \cdot \underbrace{\begin{bmatrix} u_{11} & \mathbf{u_{12}} \\ 0 & U_{22} \end{bmatrix}}_{U}$$

$$[a_{11} \; \vec{a}_{12}] = 1 \cdot [u_{11} \; \vec{u}_{12}]$$

$u_{11} = a_{11}$

$u_{12} = a_{12}$

also 1



$A_{22} =$

$\vec{l}_{21} \vec{u}_{12}^{T} + L_{22} U_{22}$

$u_{11} \cdot \vec{l}_{21} = \vec{a}_{21}$

$$\vec{l}_{21} = \vec{a}_{21} / u_{11}$$

assuming $u_{11} \neq 0$.

$$A_{22} = \vec{l}_{21} \vec{u}_{12}^{\,T} + L_{22} U_{22}$$

$$\underbrace{A_{22} - \vec{l}_{21} \vec{u}_{12}^{\,T}} = L_{22} \; U_{22}$$

a_{12}

$$\left[\begin{array}{c|ccc} 1 & 2 & 3 & 4 \\ \hline 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{array}\right]$$

a_{21}

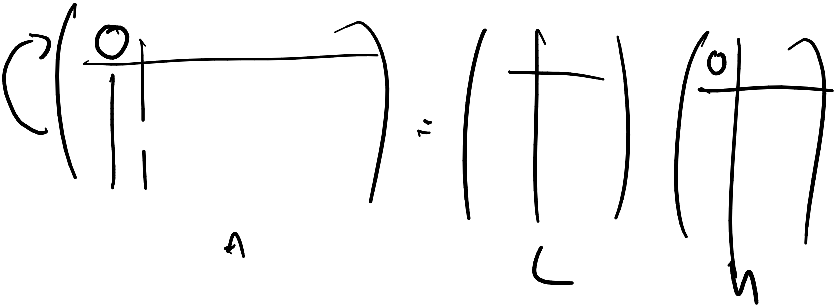
Demo: Gaussian Elimination

LU: Failure Cases?

Is LU/Gaussian Elimination bulletproof?

No, might divide by zero

What can be done to get something *like* an LU factorization?



Partial Pivoting Example

Lets try to get an pivoted LU factorization of the matrix

$$A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$$

$u_{11} = 0$

$u_{12} = 1$

$$l_{21} \cdot \underbrace{\cancel{u_{11}}}_{0} + 1 \cdot 0 = 2$$

Permutation Matrices

How do we capture 'row swaps' in a factorization?

$$\overset{P}{\begin{pmatrix} 1 & & & \\ & & 1 & \\ & 1 & & \\ & & & 1 \end{pmatrix}} \overset{A}{\begin{pmatrix} A & A & A & A \\ B & B & B & B \\ C & C & C & C \\ D & D & D & D \end{pmatrix}} = \begin{pmatrix} A & - & - \\ C & - & - \\ B & - & - \\ D & - & - \end{pmatrix}$$

Permutation mat

General LU Partial Pivoting

u_{11} ← the thing you divide by

What does the overall process look like?

1. Swap a row with a nz first entry to the top
2. Perform one step of LU as usual
3. Repeat

Computational Cost

What is the computational cost of multiplying two $n \times n$ matrices?

What is the computational cost of carrying out LU factorization on an $n \times n$ matrix?