

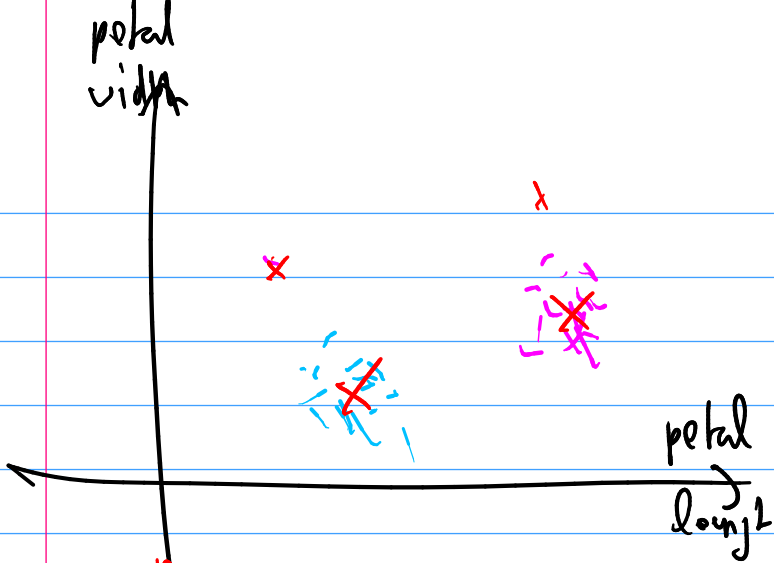
Overview

- LU

Exanlet 4

(
0)

$\begin{matrix} \text{Fi} & \text{Sa} & \text{Sn} \\ 700 & & \end{matrix}$



$$\frac{1}{n} \sum_{i=1}^n x_i$$

General LU Factorization (Gaussian Elimination)

$$A = \begin{bmatrix} a_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \mathbf{l}_{21} & L_{22} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{u}_{11} & \mathbf{u}_{12} \\ 0 & U_{22} \end{bmatrix}$$

L
↑
U

$$[a_{11} \ \vec{a}_{12}] = 1 \cdot [u_{11} \ \vec{u}_{12}]$$

$u_{11} = a_{11}$
 $u_{12} = \vec{a}_{12}$

also i ✓

1	0		
\vec{l}_{21}	L_{22}		

u_{11}	\vec{u}_{12}		
0	U_{22}		

U

$A_{22} =$

$\vec{l}_{21} \vec{u}_{12} + L_{22} U_{22}$

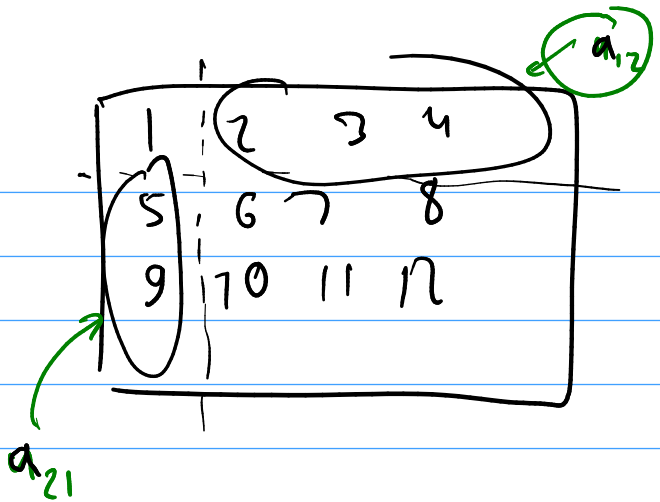
$\vec{u}_{11} \cdot \vec{l}_{21} = \vec{a}_{21} \checkmark$

$$l_{21} = a_{21} / u_{11}$$

assuming $u_{11} \neq 0$.

$$A_{22} = l_{21} u_{12} + L_{22} u_{22}$$

$$A_{22} - l_{21} u_{12} = L_{22} u_{22}$$



Demo: Gaussian Elimination

LU: Failure Cases?

Is LU/Gaussian Elimination bulletproof?

No, might divide by zero

What can be done to get something *like* an LU factorization?

The diagram illustrates the decomposition of a matrix A into a lower triangular matrix L and an upper triangular matrix U . Matrix A is shown on the left with a circled zero in the top-left corner. A horizontal line is drawn from the zero to the right, and a vertical line is drawn from the zero down, forming a cross. A curved arrow on the left points from the zero towards the top-left corner. Matrix L is shown in the middle, consisting of a vertical line with a horizontal crossbar at the top, representing a lower triangular matrix with ones on the diagonal. Matrix U is shown on the right, consisting of a vertical line with a horizontal crossbar at the top, representing an upper triangular matrix with a zero in the top-left corner. The matrices are connected by a plus sign, indicating the equation $A = L + U$.

Partial Pivoting Example

Lets try to get an pivoted LU factorization of the matrix

$$A = \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}.$$

$$\begin{pmatrix} 1 & 0 \\ l_{21} & 1 \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} \\ 0 & u_{22} \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 2 & 1 \end{pmatrix}$$

$u_{11} = 0$
 $u_{12} = 1$

$$l_{21} \cdot \cancel{u_{11}} + 1 \cdot 0 = 2$$

0

Permutation Matrices

How do we capture 'row swaps' in a factorization?

$$\begin{pmatrix} 1 & & & & \\ & 1 & & & \\ & & 1 & & \\ & & & 1 & \\ & & & & 1 \end{pmatrix} \begin{pmatrix} A & A & A & A \\ B & B & B & B \\ C & C & C & C \\ D & D & D & D \end{pmatrix} = \begin{pmatrix} A & \dots & \dots & \dots \\ C & \dots & \dots & \dots \\ B & \dots & \dots & \dots \\ D & \dots & \dots & \dots \end{pmatrix}$$

↑
Permutation mat

General LU Partial Pivoting

$u_{ij} \leftarrow$ the thing you divide by

What does the overall process look like?

1. Swap a row with a new first entry to the top
2. Perform one step of LU as usual
3. Repeat

Computational Cost

What is the computational cost of multiplying two $n \times n$ matrices?

What is the computational cost of carrying out LU factorization on an $n \times n$ matrix?