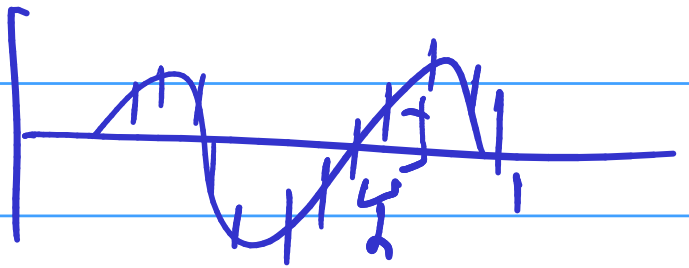
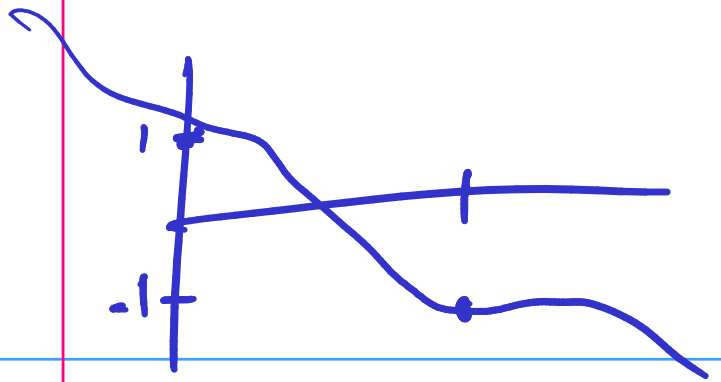
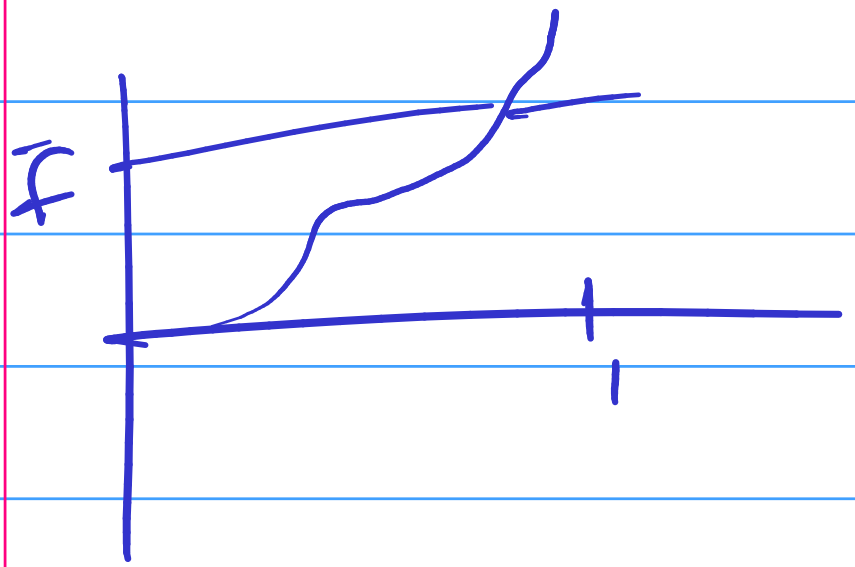
HW 4, P1

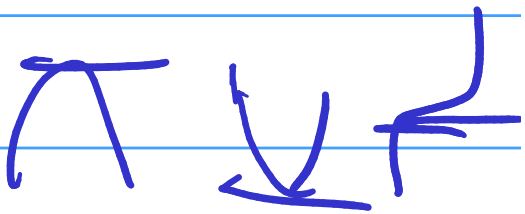
$$\kappa_{abs} = \max_{x \in [0,1]} |f'(x)|$$

$$\approx \max_{x \in [0,1]} \frac{|f(x+h) - f(x)|}{|h|}$$



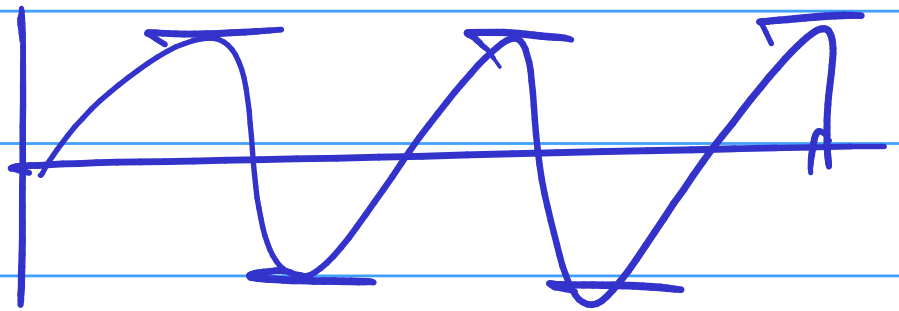






$f'(x) = 0$

x-critical point



Theme: cast your problem in linear algebra

1. high level rep. of algorithm
2. performance
3. use numerics theory

$A =$ movies (attributes)

$A[i,:] = (a_1 \dots a_m)$ (attributes prefs)

a_j is jth attribute of movie i

$X[:,i] =$ $\begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix}$

p_j is pref. for jth attribute of friend i

$\hat{Y} = \hat{A} \cdot \hat{X}$

Y_{ij} → predicted rating of friend j for movie i

$$y_i = \sum_{j=1}^{n} Y_{ij}$$

y - overall rating

A = movie index $\begin{pmatrix} \text{attribute} \\ \text{attributes} \end{pmatrix}$

$Y = AX$

$X = A^{-1}Y$

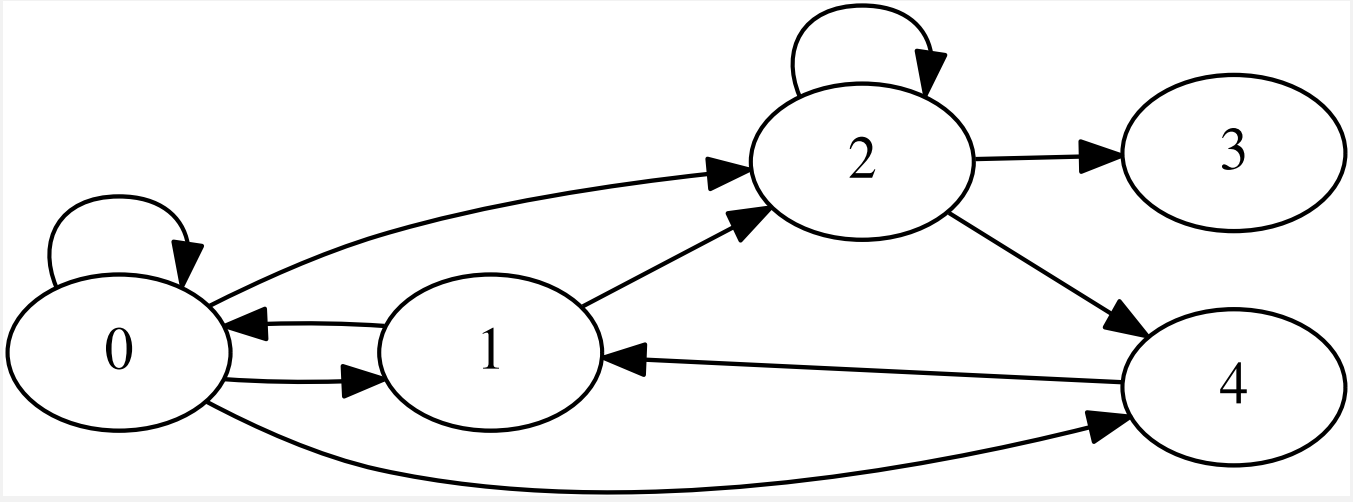
X = attribute pref. $\begin{pmatrix} \text{friend} \\ \text{preferences} \end{pmatrix}$

Y = movie $\begin{pmatrix} \text{friend} \\ \text{ratings} \end{pmatrix}$

Outline

Graphs as Matrices

How could this (directed) graph be written as a matrix?



$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Matrices for Graph Traversal: Technicalities

What is the general rule for turning a graph into a matrix?

$A_{ji} = 1$ if there is an edge $i \to j$

What does the matrix for an *undirected* graph look like?

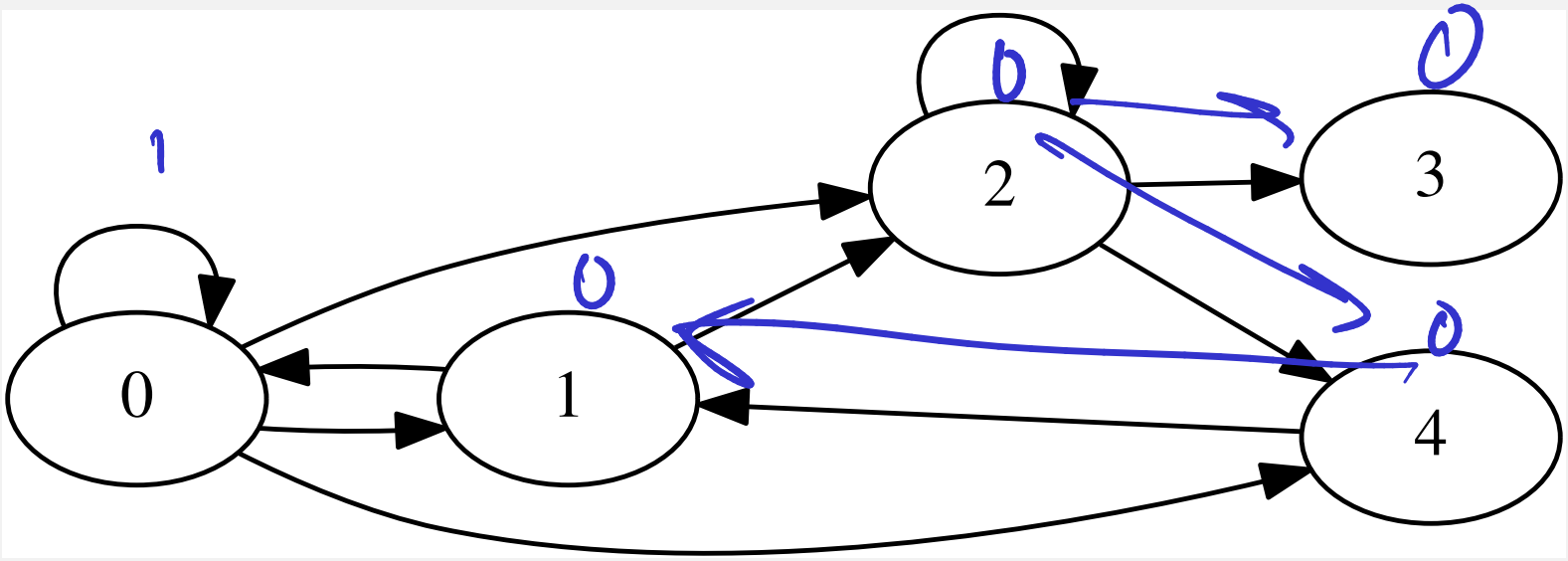
symmetric

How could we turn a *weighted graph* (i.e. one where the edges have weights–maybe 'pipe widths') into a matrix?

puts weights into correspondig matrix entries

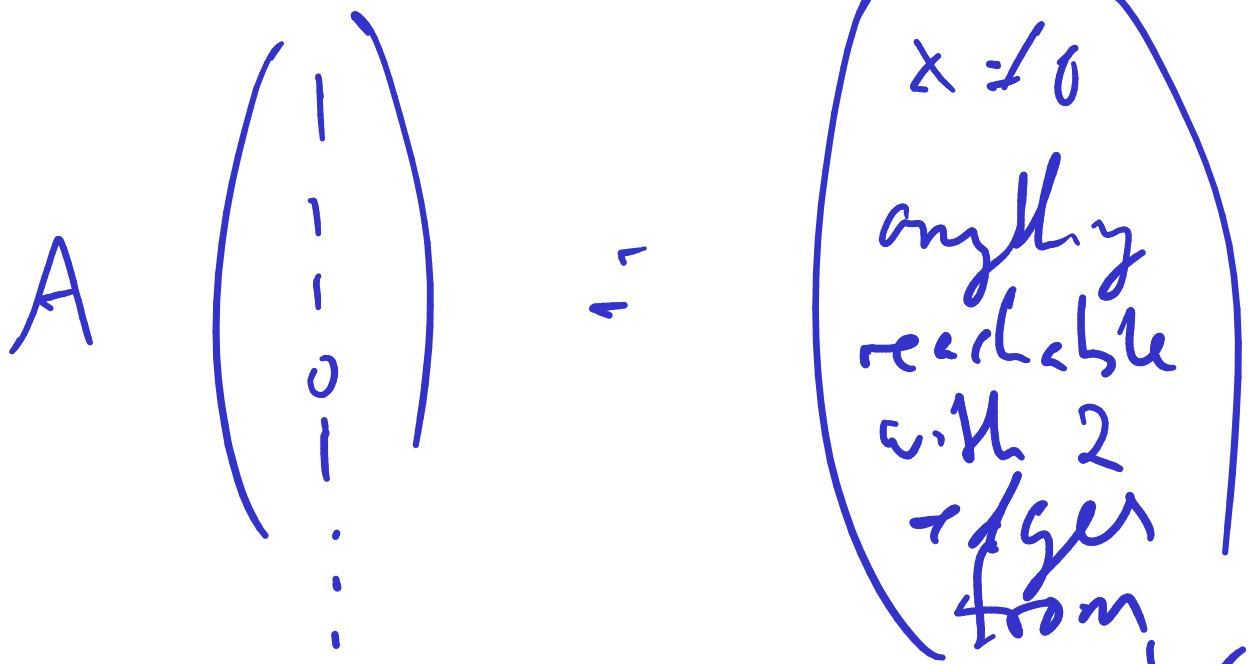
Graph Matrices and Matrix-Vector Multiplication

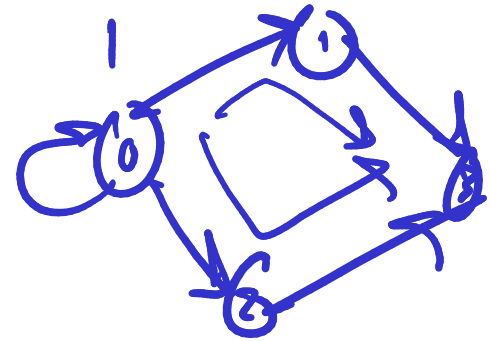
If we multiply a graph matrix by the ith unit vector, what happens?



$$A \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 1 \end{pmatrix} \qquad A \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Demo: Matrices for Graph Traversal





B

$B\begin{pmatrix}0\\0\\1\\0\end{pmatrix} = B(Bx)$

$x = \begin{pmatrix}1\\0\\0\\0\end{pmatrix}$

Outline