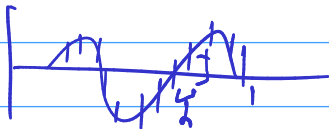


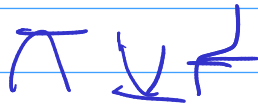
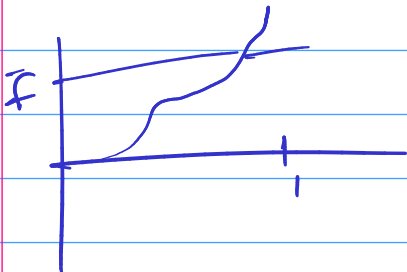
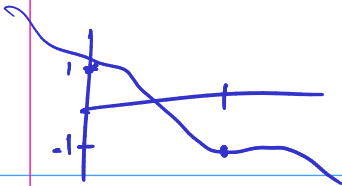
HW 4, P 1

$$\kappa_{abs} = \max_{x \in [0,1]} |f'(x)|$$

$$\approx \max_{x \in [0,1]}$$

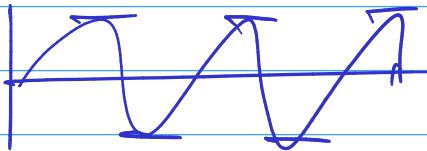
$$\frac{|f(x+h) - f(x)|}{|h|}$$





$$f'(x) = 0$$

x-critical
point



Theme: cast your problem
in linear algebra

1. high level rep. of algorithm

2. performance

3. use numerics theory

$$A = \text{movies} \begin{pmatrix} \text{attributes} \end{pmatrix} \begin{pmatrix} \end{pmatrix}$$

attributes
prefs

$$A[i, :] = (a_1 \dots a_m)$$

a_j is j th attribute of movie i

$$X[:, i] = \begin{pmatrix} p_1 \\ \vdots \\ p_m \end{pmatrix}$$

p_j is pref.
for j th attribute
of friend i

$$\underline{\underline{K \hat{Y} = K \hat{A} \cdot \hat{X}}}$$

Y_{ij} → predicted rating
of friend j for movie i

$$y_i = \sum_{j=1}^n V_{ij}$$

y - overall rating

$$Y = AX$$
$$X = A^{-1}Y$$

A = movie index (attribute
attributes)

X = attribute pref. (friend
preferences)

Y = movie (friend
ratings)

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Python, Numpy, and Matplotlib
Making Models with Polynomials
Making Models with Monte Carlo

Error, Accuracy and Convergence
Floating Point

Modeling the World with Arrays

The World in a Vector
What can Matrices Do?
Graphs
Sparsity

Norms and Errors
The 'Undo' Button for Linear Operations: LU

LU: Applications
Linear Algebra Applications
Interpolation

Repeating Linear Operations:
Eigenvalues and Steady States
Eigenvalues: Applications
Approximate Undo: SVD and Least Squares

SVD: Applications

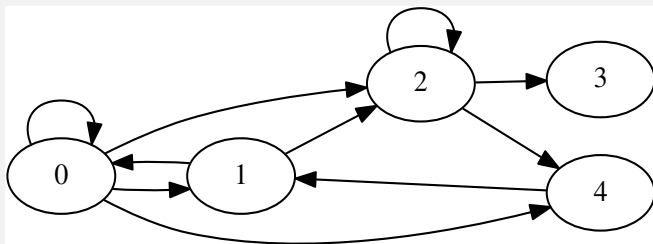
Solving Funny-Shaped Linear Systems
Data Fitting
Norms and Condition Numbers
Low-Rank Approximation

Iteration and Convergence

Solving One Equation
Solving Many Equations
Finding the Best: Optimization in 1D
Optimization in n Dimensions

Graphs as Matrices

How could this (directed) graph be written as a matrix?



$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \end{bmatrix}$$

Matrices for Graph Traversal: Technicalities

What is the general rule for turning a graph into a matrix?

$A_{ji} = 1$ if there is an edge $i \rightarrow j$

What does the matrix for an *undirected* graph look like?

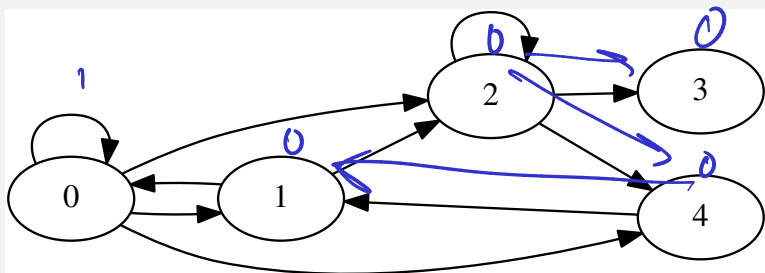
symmetric

How could we turn a *weighted graph* (i.e. one where the edges have weights—maybe 'pipe widths') into a matrix?

puts weights into corresponding matrix entries

Graph Matrices and Matrix-Vector Multiplication

If we multiply a graph matrix by the i th unit vector, what happens?



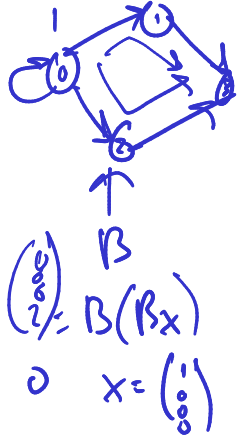
$$A \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad \bigg| \quad A \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Demo: Matrices for Graph Traversal

$$A \begin{pmatrix} | \\ | \\ | \\ 0 \\ | \\ \vdots \end{pmatrix}$$

\rightarrow

$x \neq 0$
 anything
 reachable
 with 2
 edges
 from
 vertex $x = 0$



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