Quadrature weights

$\underbrace{d \cdot V}_{w \text{ - weights}}$ $\longrightarrow$ Vandermonde matrix with basis evaluated nodes

integrals of basis functions

Outline

Solving Nonlinear Equations

What is the goal here?

given f

find $f(\underset{\uparrow}{x}) = 0$

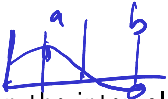
$f(x) = g$

$\tilde{f}(x) = f(x) - g$

Solve $\tilde{f}(\underset{\uparrow}{x}) = 0$

Bisection Method

Assume continuos function f has a zero on the interval $[a, b]$ and

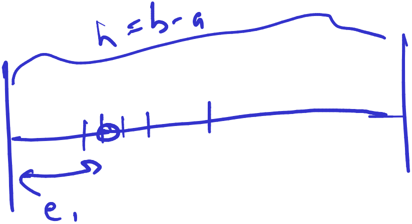


$$\text{sign}(f(a)) = -\text{sign}(f(b)).$$

Perform binary search: check sign of $f((a+b)/2)$ and define new search interval so that ends have opposite sign.

Demo: Bisection Method

> What's the rate of convergence? What's the constant?



$e_1 \le h$

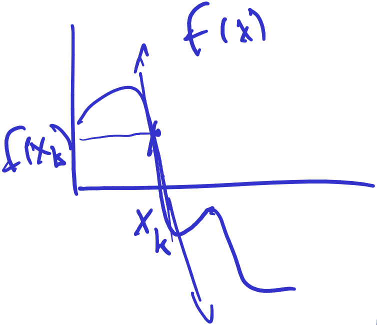
$e_1 \le h/2$

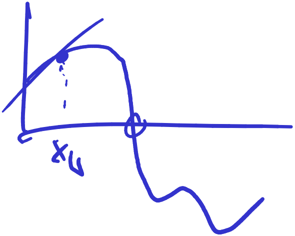
$e_2 \le h/4$

linear

Newton's Method

Derive Newton's method.





$$f(x) \approx \underbrace{f(x_k) + f'(x_k)(x-x_k)}_{\tilde{f}_k(x)}$$

$$\tilde{f}_k(x) = 0$$

$$\Rightarrow \quad 0 = f(x_k) + f'(x_k)(x - x_k)$$

$$0 = f(x_k) + f'(x_k)\,x - f'(x_k)\,x_k$$

$$x = \frac{-f(x_k)}{f'(x_k)} + x_k$$

$$x = x_k - \frac{f(x_k)}{f'(x_k)}$$

x_{k+1}

Demo: Newton's method
Demo: Convergence of Newton's Method

What are some **drawbacks** of Newton?

may not converge
stuck at local minima
or move far away from 1st
guess

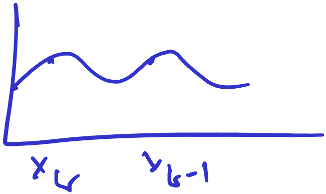
need derivative

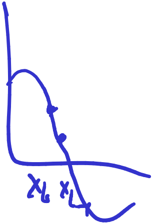
Secant Method

What would Newton without the use of the derivative look like?

$$f'(x_k) \approx \frac{f(x_k) - f(x_{k-1})}{x_k - x_{k-1}}$$

want $x_k \approx x_{k-1}$





e_k = error at iter k

Secant $\rightarrow$ $$\lim_{k \to \infty} \frac{e_{k+1}}{e_k e_{k-1}} = C$$

$$\lim_{k \to \infty} \frac{e_{k+1}}{e_k^2} = C$$

Side Note

Secant Method Drawbacks

What are some **drawbacks** of Secant?

- 2 starting guesses
- convergence problem Newton

Demo: Secant Method

In-class activity: Secant Method

+ no derivative

Outline

Solving Nonlinear Equations

What is the goal here?

$$\vec{f}(\vec{x}) = \vec{0}$$

$$f(\vec{x}) = \vec{b}$$

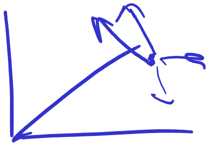
$$\tilde{f}(\vec{x}) = f(\vec{x}) - \vec{b}$$

$$f(\vec{x}) = \begin{bmatrix} f_1(\vec{x}) \\ f_2(\vec{x}) \\ \vdots \\ f_n(\vec{x}) \end{bmatrix} = \begin{bmatrix} f_1(x_1, \ldots, x_n) \\ \vdots \\ f_n(x_1, \ldots, x_n) \end{bmatrix}$$

Newton's method

What does Newton's method look like in n dimensions?

$$J_f(x) = \begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x) & \cdots & \frac{\partial f_1}{\partial x_n}(x) \\ \vdots & \ddots & \vdots \\ \frac{\partial f_n}{\partial x_1}(x) & \cdots & \frac{\partial f_n}{\partial x_n}(x) \end{bmatrix}$$



$$x_{k+1} = x_k - J_f(x_k)^{-1} f(x_k)$$