Computing the SVD (Kiddy Version)

How can I compute an SVD of a matrix A?

$$A = U\Sigma V^T \Rightarrow A^TA = V\Sigma \underline{U^TU}\Sigma V^T = V\Sigma^2 V^T$$

1

$$\Rightarrow \underline{A^TA}\, V = V\Sigma^2 \underline{V^TV} = V\Sigma^2$$

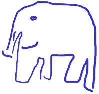
$$\underline{U} = A(\Sigma V^T)^{-1} = AV\Sigma^{-1}$$

2

3

Outline

Solving Nonlinear Equations



unknowns

equations

What is the goal here?

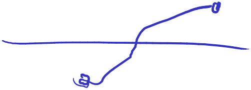
$$f: \mathbb{R}^n \to \mathbb{R}^n$$

$$f(\vec{x}) = \vec{0}$$

$$g(\vec{x}) = h(\vec{x})$$

Showing Existence

How can we show existence of a root?



- Intermediate value thm.
- Inverse Function theorem

  If $J_F = f'$ invertible at x, then there exists a nbh around x so that f is invertible function $f^{-1}(0)$

- Contraction mapping theorem

  g is contractive if there exists a $0<\gamma<1$ so that $\|g(x)-g(y)\| \le \gamma \|x-y\|$

  there exists a x so that $g(x)=x$ (on a closed set X)

In general: no uniqueness results

Sensitivity and Multiplicity

What is the sensitivity/conditioning of root finding?

$$\operatorname{cond}(\text{root finding}) = \operatorname{cond}(\text{evaluation of } f^{-1}(0))$$

What are multiple roots?

$$0 = f(x) = f'(x) = \cdots = f^{(m-1)}(x)$$

root of multiplicity m

How do multiple roots interact with conditioning?

The inverse function is steep $\Rightarrow$ cond is poor.

In-Class Activity: Krylov and Nonlinear Equations

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Rates of Convergence

What is *linear convergence*? *quadratic convergence*?

$e_k = \hat{u}_k - u$, $e_k \to 0$ as $k \to \infty$

$$\lim_{k\to\infty} \frac{\|e_{k+1}\|}{\|e_k\|^r} = c \in (0, +\infty)$$

- $r = 1$ linear. (power iteration)
- $r > 1$ superlinear
- $r = 2$ quadratic (Rayleigh quotient iter)

About Convergence Rates

Demo: Rates of Convergence [cleared]

Characterize linear, quadratic convergence in terms of the 'number of accurate digits'.