

Computing the SVD (Kiddy Version)

How can I compute an SVD of a matrix A ?

$$A = U \Sigma V^T \Rightarrow A^T A = V \Sigma \underline{U^T U} \Sigma V^T = V \Sigma^2 V^T$$
$$\Rightarrow \textcircled{1} \underline{A^T A} V = V \Sigma^2 \underline{V^T V} = V \Sigma^2$$
$$\underline{U} \textcircled{3} = A (\underline{\Sigma V^T})^{-1} = A V \underbrace{\Sigma^{-1}} \textcircled{2}$$

Outline

Introduction to Scientific Computing

Systems of Linear Equations

Linear Least Squares

Eigenvalue Problems

Nonlinear Equations

Introduction

Iterative Procedures

Methods in One Dimension

Methods in n Dimensions ("Systems of Equations")

Optimization

Interpolation

Numerical Integration and Differentiation

Initial Value Problems for ODEs

Boundary Value Problems for ODEs

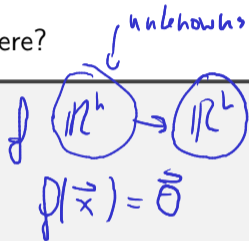
Partial Differential Equations and Sparse Linear Algebra

Fast Fourier Transform

Additional Topics

Solving Nonlinear Equations

What is the goal here?



$$g(\vec{x}) = h(\vec{x})$$

Showing Existence

How can we show existence of a root?



- Intermediate value thm.
 - Inverse Function theorem
If $\exists f = f'$ invertible ^{at x} , then there exists a nbh around x so that f is invertible function $f^{-1}(0)$
 - Contraction mapping theorem
 g is contractive if there exists a $0 < \gamma < 1$ so that $\|g(x) - g(y)\| \leq \gamma \|x - y\|$
there exists a x so that $g(x) = x$ (on closed set X)
- In general: no uniqueness results

Sensitivity and Multiplicity

What is the sensitivity/conditioning of root finding?

$$\text{cond}(\text{root finding}) = \text{cond}(\text{evaluation of } f'(0))$$

What are multiple roots?

$$0 = f(x) = f'(x) = f^{(m-1)}(x)$$

root of multiplicity m



How do multiple roots interact with conditioning?

The inverse function is steep \Rightarrow cond is poor.

In-Class Activity: Krylov and Nonlinear Equations

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Rates of Convergence

What is *linear convergence*? *quadratic convergence*?

$$e_k = \hat{u}_k - u, \quad e_k \rightarrow 0 \text{ as } k \rightarrow \infty$$

$$\lim_{k \rightarrow \infty} \frac{\|e_{k+1}\|}{\|e_k\|^r} = c \in (0, +\infty)$$

- $r = 1$ linear. (power iteration)
- $r > 1$ superlinear
- $r = 2$ quadratic (Rayleigh quotient iter)

About Convergence Rates

Demo: Rates of Convergence [cleared]

Characterize linear, quadratic convergence in terms of the 'number of accurate digits'.

