Stability and Accuracy

Previously: Considered *problems* or *questions*.

Next: Considered *methods*, i.e. computational approaches to find solutions.

When is a method *accurate*?

Closeness of the method output to the true answer

When is a method *stable*?

- A method is stable if it produces the exact answer to a "nearby" problem
  - "backward stability" stricter than the simpler requirement
    Sensitivity of the numerical method to variation in input is no (or not much) greater than the conditioning of the underlying prob.

Getting into Trouble with Accuracy and Stability

bitly / cs450-f22

How can I produce inaccurate results?

- apply a method that's inaccurate
- try to compute something ill-conditioned
- try to compute something well-conditioned but use an unstable method.

In-Class Activity: Forward/Backward Error

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Wanted: Real Numbers... in a computer

Computers can represent *integers*, using bits:

$$23 = 1 \cdot 2^4 + 0 \cdot 2^3 + 1 \cdot 2^2 + 1 \cdot 2^1 + 1 \cdot 2^0 = (10111)_2$$

How would we represent fractions?

$23.625 = \ldots + 1 \cdot 2^0$

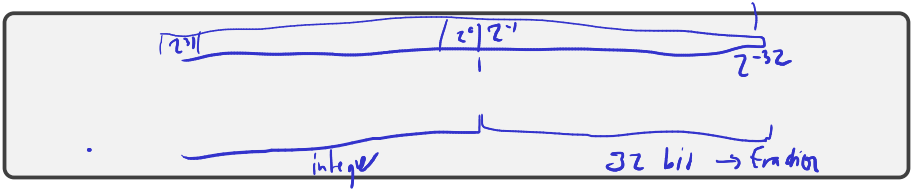
$1 \cdot 2^{-1} + 0 \cdot 2^{-2} + 1 \cdot 2^{-3}$

$= (10111.101)_2$

'fixed point'

Fixed-Point Numbers

Suppose we use units of 64 bits, with 32 bits for exponents $\geqslant 0$ and 32 bits for exponents < 0. What numbers can we represent?



How many 'digits' of relative accuracy (think relative rounding error) are available for the smallest vs. the largest number?

rel. error of 2^{-64} : about 19 digits

For smallest: 0

Floating Point Numbers

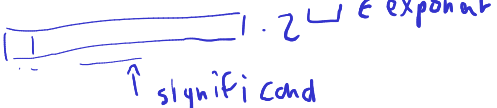
Convert $13 = (1101)_2$ into floating point representation.

$(1.101)_2 \cdot 2^3$

What pieces do you need to store an FP number?

significand: $(1.101)_2$
exponent: 3

$4.35 \cdot 10^{70}$

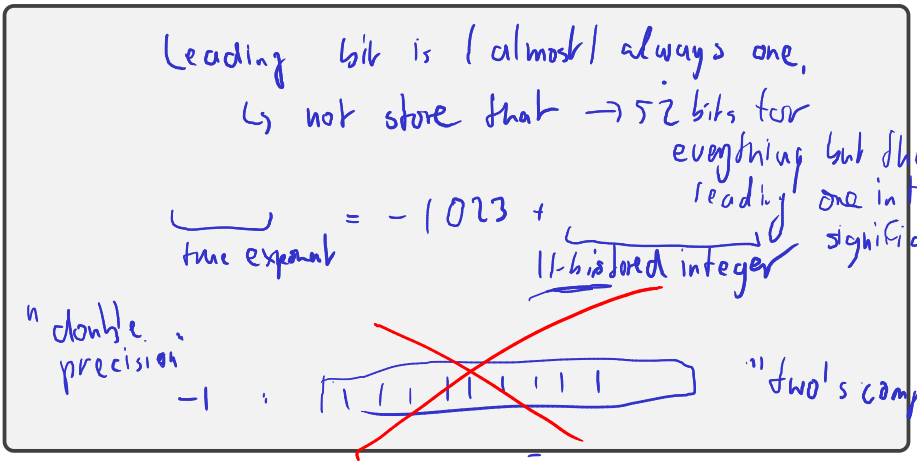


Floating Point: Implementation, Normalization

Previously: Consider *mathematical* view of FP. (via example: $(1.101)_2$)

Next: Consider *implementation* of FP in hardware.

Do you notice a source of inefficiency in our number representation?



Unrepresentable numbers?

Can you think of a somewhat central number that we cannot represent as

$$x = (1.\_\_\_\_\_\_\_\_\_\_)_2 \cdot 2^{-p}?$$

Zero is not yet representable

"If stored exponent bit pattern is 000...000, turn off the leading one"

... and do what with the exponent?

Demo: Picking apart a floating point number [cleared]

Subnormal Numbers

What is the smallest representable number in an FP system with 4 stored bits in the significand and a stored exponent range of $[-7, 8]$?