

- Examenlet 3
- Quiz 16  $\rightarrow$  EC

$$Q_u R_u = X_u - \sigma I$$

$$X_{u+1} = R_u Q_u + \sigma I$$

$$S_u = \text{span}(\vec{x}_0, A\vec{x}_0, A^2\vec{x}_0, \dots, A^k\vec{x}_0)$$

$$\left. \begin{array}{l} A: \mathbb{R}^k \rightarrow \mathbb{R}^k \\ A|_{S_u}: S_u \rightarrow S_u \end{array} \right\}$$



## Conditioning in Krylov Space Methods/Arnoldi Iteration (II)

$$Q_n^T A Q_n = H$$

↳ assumes the role of reduction  
to upper Hessenberg

(From Householder similarity  
transform)

Demo: Arnoldi Iteration [cleared] (Part 1)

# Krylov: What about eigenvalues?

How can we use Arnoldi/Lanczos to compute eigenvalues?

$$Q = Q_n = \begin{bmatrix} Q_k & U_k \end{bmatrix}$$

known                      unknown

$$H = Q^T A Q = \begin{bmatrix} Q_k \\ U_k^T \end{bmatrix} A \begin{bmatrix} Q_k & U_k \end{bmatrix} = \begin{bmatrix} \text{blue dots} & \text{orange dots} \\ \text{orange dots} & \text{orange dots} \end{bmatrix}$$

$H$

Arnoldi iteration

For symm: Lanczos iteration       $A$

# Computing the SVD (Kiddy Version)

$$A = U \Sigma V^T$$

1. Compute eigenvalues / eigenvectors of  $A^T A$ :

$V$  is often become  
eigenvect. of symm.  $A^T A$

$$A^T A V = V D$$

$\Sigma$  can't be computed accurately

$$V^T A^T A V = D = \begin{cases} \Sigma^2 \\ \Sigma \geq 0 \end{cases}$$

2.

$$U \Sigma = A V$$

IF  $\Sigma$  is invertible:  $U = A V \Sigma^{-1}$

$$U^T U = \Sigma^{-1} V^T A^T A V \Sigma^{-1} = \Sigma^{-1} \Sigma^2 \Sigma^{-1} = I$$

## Demo: Computing the SVD [cleared]

"Actual"/"non-kiddy" computation of the SVD:

► Bidiagonalize  $A = U \begin{bmatrix} B \\ 0 \end{bmatrix} V^T$ , then diagonalize via variant of QR.

► References: [Chan '82](#) or Golub/van Loan Sec 8.6.



# Outline

Introduction to Scientific Computing

Systems of Linear Equations

Linear Least Squares

Eigenvalue Problems

**Nonlinear Equations**

Introduction

Iterative Procedures

Methods in One Dimension

Methods in  $n$  Dimensions ("Systems of Equations")

Optimization

Interpolation

Numerical Integration and Differentiation

Initial Value Problems for ODEs

Boundary Value Problems for ODEs

Partial Differential Equations and Sparse Linear Algebra

Fast Fourier Transform

Additional Topics

# Solving Nonlinear Equations

What is the goal here?

$$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

Solve for  $\vec{f}(\vec{x}) = \vec{0}$  ← "WLOG" RHS is zero,  
if not, absorb into  $\vec{f}$

## Showing Existence

How can we show existence of a root?

