- Examlet 3
- Quiz 16 → EC

$$Q_k R_k = X_k - \sigma I$$

$$X_{k+1} = R_k Q_k + \sigma I$$

$$S_k = \text{span}(\vec{x}_0, A\vec{x}_0, A^2\vec{x}_0, \dots, A^k\vec{x}_0)$$

$$A : \mathbb{R}^n \to \mathbb{R}^n$$

$$\hookrightarrow A|_{S_k} : S_k \to S_k$$

Conditioning in Krylov Space Methods/Arnoldi Iteration (II)

$Q_n^T A Q_n = H$

↳ assumes the role of reduction to upper Hessenberg (from Householder similarity transform)

Demo: Arnoldi Iteration [cleared] (Part 1)

Krylov: What about eigenvalues?

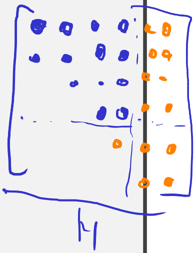
How can we use Arnoldi/Lanczos to compute eigenvalues?

$$Q = Q_n = [\,Q_k \;\vdots\; U_k\,]$$

known

unknown

$$H = Q^T A Q = \begin{bmatrix} Q_k \\ U_k^T \end{bmatrix} A \,[\,Q_k \;\vdots\; U_k\,] =$$



Arnoldi iteration

For symm: Lanczos iteration A

Demo: Arnoldi Iteration [cleared] (Part 2)

Computing the SVD (Kiddy Version)

$A = U\Sigma V^T$

1. Compute eigenvalues/eigenvectors of A^TA:

$$\underbrace{A^TA}_{\text{can't be computed accurately}}V = VD. \qquad V^TA^TAV = D =: \Sigma^2 \qquad \Sigma \geq 0$$

V is orth because eigenvec. of symm. A^TA

2. $U\Sigma = AV$

If Σ is invertible: $U = AV\Sigma^{-1}$

$$U^TU = \Sigma^{-1}V^TA^TAV\Sigma^{-1} = \Sigma^{-1}\Sigma^2\Sigma^{-1} = I$$

Demo: Computing the SVD [cleared]

"Actual"/"non-kiddy" computation of the SVD:

$B =$

- Bidiagonalize $A = U\begin{bmatrix} B \\ 0 \end{bmatrix}V^T$, then diagonalize via variant of QR.
- References: Chan '82 or Golub/van Loan Sec 8.6.

Outline

Solving Nonlinear Equations

What is the goal here?

$f: \mathbb{R}^n \to \mathbb{R}^n$

Solve for $\vec{f}(\vec{x}) = \vec{0}$

"WLOG" RHS is zero, if not, absorb into $\vec{F}$

Showing Existence

How can we show existence of a root?