

Announcements

- ▶ HW10 due Dec 18
 - ▶ If current planning holds up, problem 2 may be useful for the final
- ▶ Final starts Thursday
 - ▶ Up to, but not including Richardson
- ▶ Please fill out ICES (will take a few minutes at end)
 - ▶ Helps me justify effort I put towards CS450

break steel

& pm Wed

Outline

Fundamentals

Systems of Linear Equations

Linear Least Squares

Eigenvalue Problems

Nonlinear Equations

Optimization

Interpolation

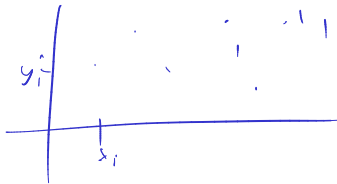
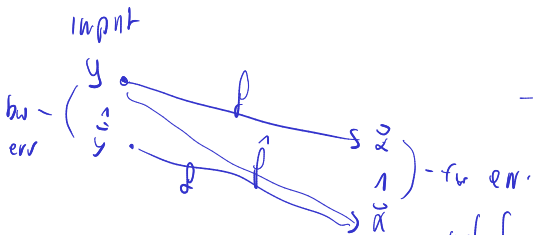
Numerical Integration and Differentiation

What does backward stability of the linear solve mean for interpolation?

$$V \vec{\alpha} = \vec{y}$$

\swarrow outputs \swarrow input
 \uparrow
 $\text{cond}(V) = \text{bad } 10^6$

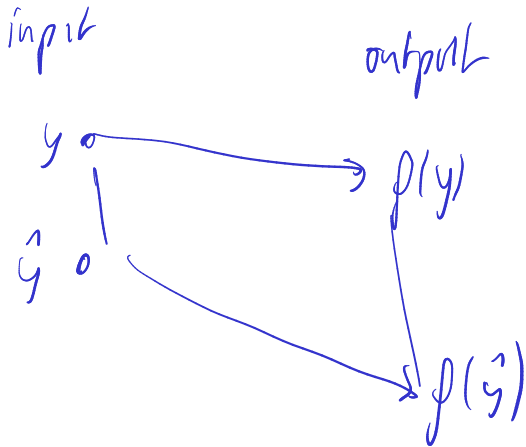
↳ nasty e.g. Gauss w/ P.P.
 $p_{n-1}(x) = \sum \alpha_i \varphi_i(x)$



$$\text{rel fw err.} = \frac{\|\hat{\alpha} - \alpha\|}{\|\alpha\|}$$

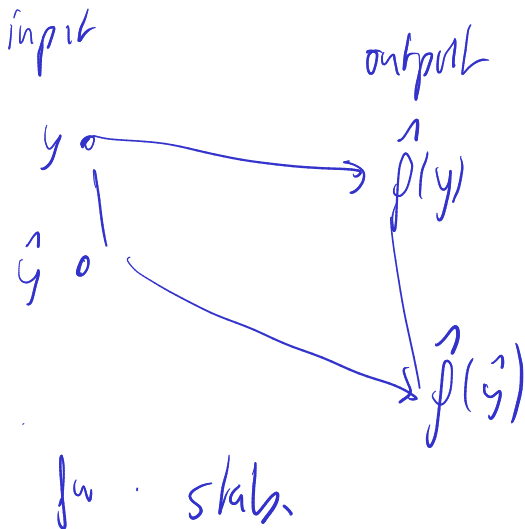
$$\text{fw}_{\text{rel abs}} \leq K_{\text{rel abs}} \cdot \text{bw}_{\text{rel abs}}$$

What does backward stability of the linear solve mean for interpolation?

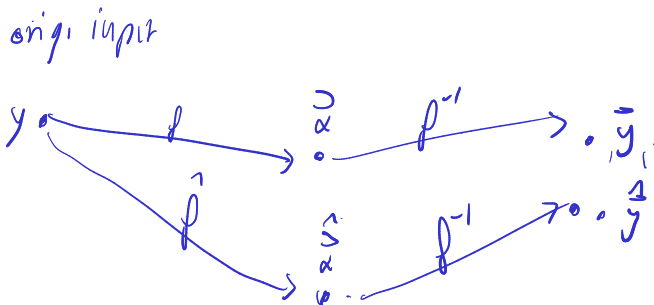


cond.

What does backward stability of the linear solve mean for interpolation?



What does backward stability of the linear solve mean for interpolation?



$$|y - \hat{y}| \leq \dots$$

What does backward stability of the linear solve mean for interpolation?

backward err. in lin. system solve:

"residual"

Forward stability:

fw perturbation α

< "stab" input perturbation β

can be obtained from
bw or fwd chains w/cord.

What does backward stability of the linear solve mean for interpolation?

backward stability:

What does backward stability of the linear solve mean for interpolation?

Name every thing Gauss!

- Gauss - Vert
- Gauss elim
- Gauss Jordan
- Gauss quad.

To get an equivalent increase in the range of magnitudes provided by subnormals, how many more exp. bits?

DP.

+ 52 stored

(53)

+ 11 exp

+ 1 sign

{ 1024 \in high spec'd
- 1022 \in spec'd

2^{52} magnitude due to no

\rightarrow less than 1

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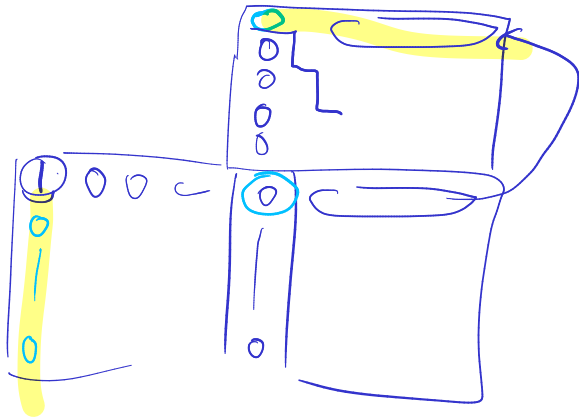
Nonlinear Equations

Optimization

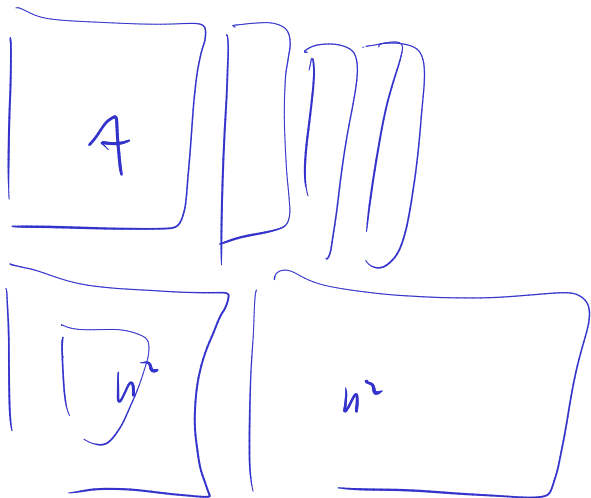
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What happens with zero columns in LU factorization?



Why is BLAS3 generically more efficient than BLAS2?



How can you use the Schur complement to lower the complexity of a solve?

↳ Woodbury.

$$\left(\begin{array}{cc|c} A & B & b_1 \\ C & D & b_2 \end{array} \right)$$

$$\left(\begin{array}{c|c} & b_1 \\ \hline D - CA^{-1}B & \dots \end{array} \right)$$

$\uparrow \quad \uparrow \quad \uparrow$

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$$A + UV^T$$

Orthogonality can only be determined up to $\sqrt{\text{eps}_{\text{mach}}}$. Why?

$$\begin{pmatrix} 1 \\ \delta \\ -1 \end{pmatrix} \quad \begin{pmatrix} 1 \\ \delta \\ 1 \end{pmatrix}$$

$$\delta < \sqrt{\epsilon_{\text{mach}}}$$

$$\delta^2 < \epsilon_{\text{mach}}$$

$$\underbrace{1 + \delta^2}_{\approx 1} - 1$$

Orthogonal matrices have perfect conditioning. What's the mismatch?

Q

$$\|Q\| = 1$$

$$\|Q^T\| = 1$$

$$\kappa_2(Q) = 1$$

$$Q^T x = \begin{pmatrix} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{pmatrix} \rightarrow \text{rel. to } \|Q^T x\| = \|x\|$$

Provide a bound on the quality of QR factorization

$$Q_{\hat{A}} \quad \dots \quad Q, A = R$$

\uparrow \uparrow

$\kappa_2 = 1$ $\kappa_2 \approx 1$

$$\|R - \hat{R}\|_2 \leq \kappa_2 \|A - \hat{A}\|_2$$

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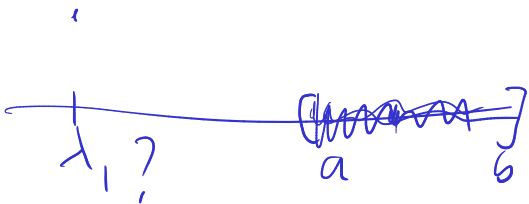
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In power iteration on a symmetric matrix, how might you 'kill off' eigenvalues in a whole interval?

- ▶ What is a "filter polynomial"?



$$A^5 - 5A^3 + A - I$$

What does a Krylov space buy you? Does it solve any specific problem?

- ▶ What's the setting for Krylov space methods?

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What does it mean when the Jacobian you encounter in Newton's method is block-diagonal?

- ▶ What is the approximant to the function used by Broyden?

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- ▶ Does convex imply coercive?
- ▶ Does coercive imply convex?
- ▶ Does unimodal imply convex?

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- ▶ What does a Lebesgue constant of 10,000 mean?

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- ▶ How do you apply finite differences on an equispaced grid given a differentiation matrix on an equispaced grid?

▶ How do you compute a composite Gaussian integral?