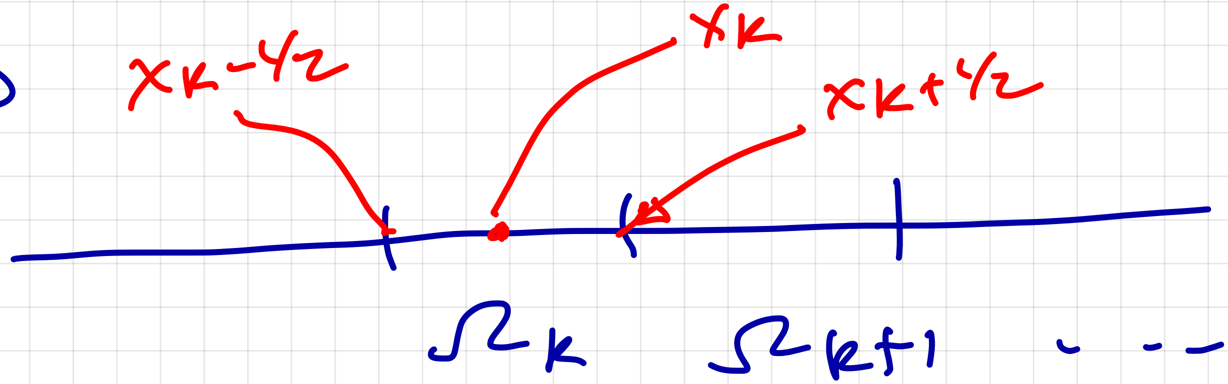
Today

- High-order FV methods

- Recap

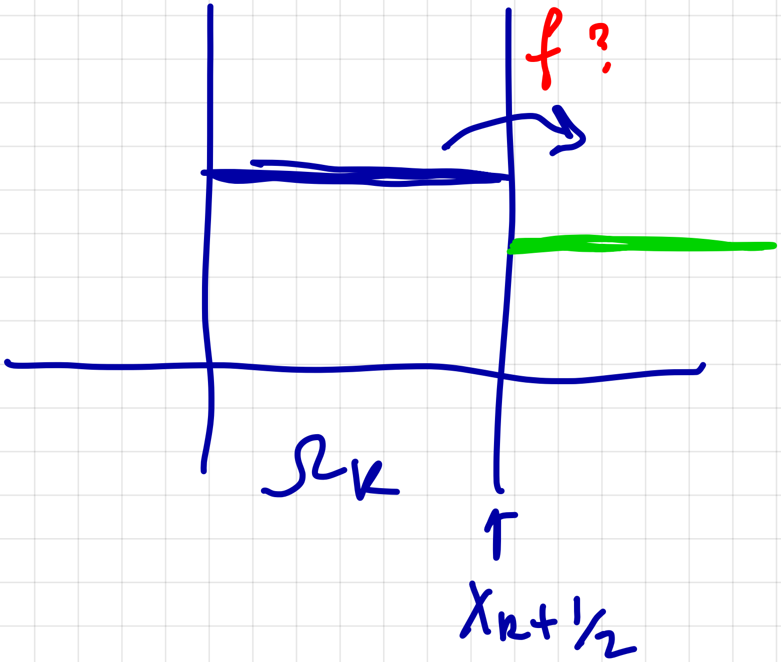


let $u_{k,\ell} \approx \overline{u}_k(t_\ell)$

average in cell Ω_k

let $\hat{f}_{k+1/2}(t)$ be the numerical flux

We look $\hat{f}_{k+1/2}(t) = f^*\left(u^-_{k+1/2},\ u^+_{k+1/2}\right)$



$u^-_{k+1/2}$ = value at $x_{k+1/2}$ "from the left"

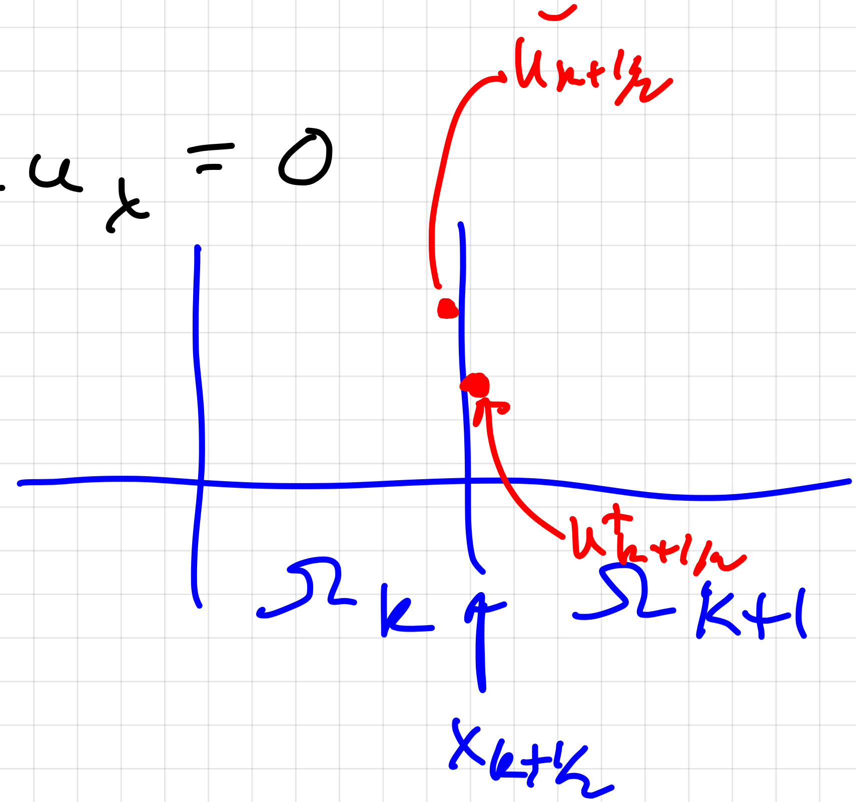
$u^+_{k+1/2}$ = value at $x_{k+1/2}$ "from the right"

Example $a > 0$

$$u_t + a u_x = 0$$

$$u^-_{k+1/2,l} = u_{kl}$$

$$u^+_{k+1/2,l} = u_{k+1,l}$$



Example $a \in \mathbb{R}$

$$f^*_{k+1/2} = \frac{a u_{kl} + a \cdot u_{k+1l}}{2} - \frac{|a|}{2}\left(u_{k+1l} - u_{kl}\right)$$

what about

$$u_t + (f(u))_x = 0 \ ?$$

$$u_t + \underbrace{f'(u)}_{\text{characteristic curve}} u_x = 0$$

FOU: $f^*(u_{k,\ell}, u_{k+1,\ell}) = \dfrac{a u_{k\ell} + a u_{k+1,\ell}}{2} - \dfrac{|a|}{2}(u_{k+1,\ell} - u_{k\ell})$

Lax-Friedrichs: $f^*(u_{k\ell}, u_{k+1,\ell}) = \dfrac{f(u_{k\ell}) + f(u_{k+1,\ell})}{2} - \dfrac{\alpha_{k+1/2}}{2}(u_{k+1,\ell} - u_{k\ell})$

$$\boxed{\frac{u_{k,\ell+1} - u_{k,\ell}}{h_t} + \frac{f^*_{k+\frac{1}{2}} - f^*_{k-\frac{1}{2}}}{h_x} = 0,}$$

$$\alpha_{k+1/2} = \max\left(|f'(u_{k\ell})|, |f'(u_{k+1,\ell})|\right)$$

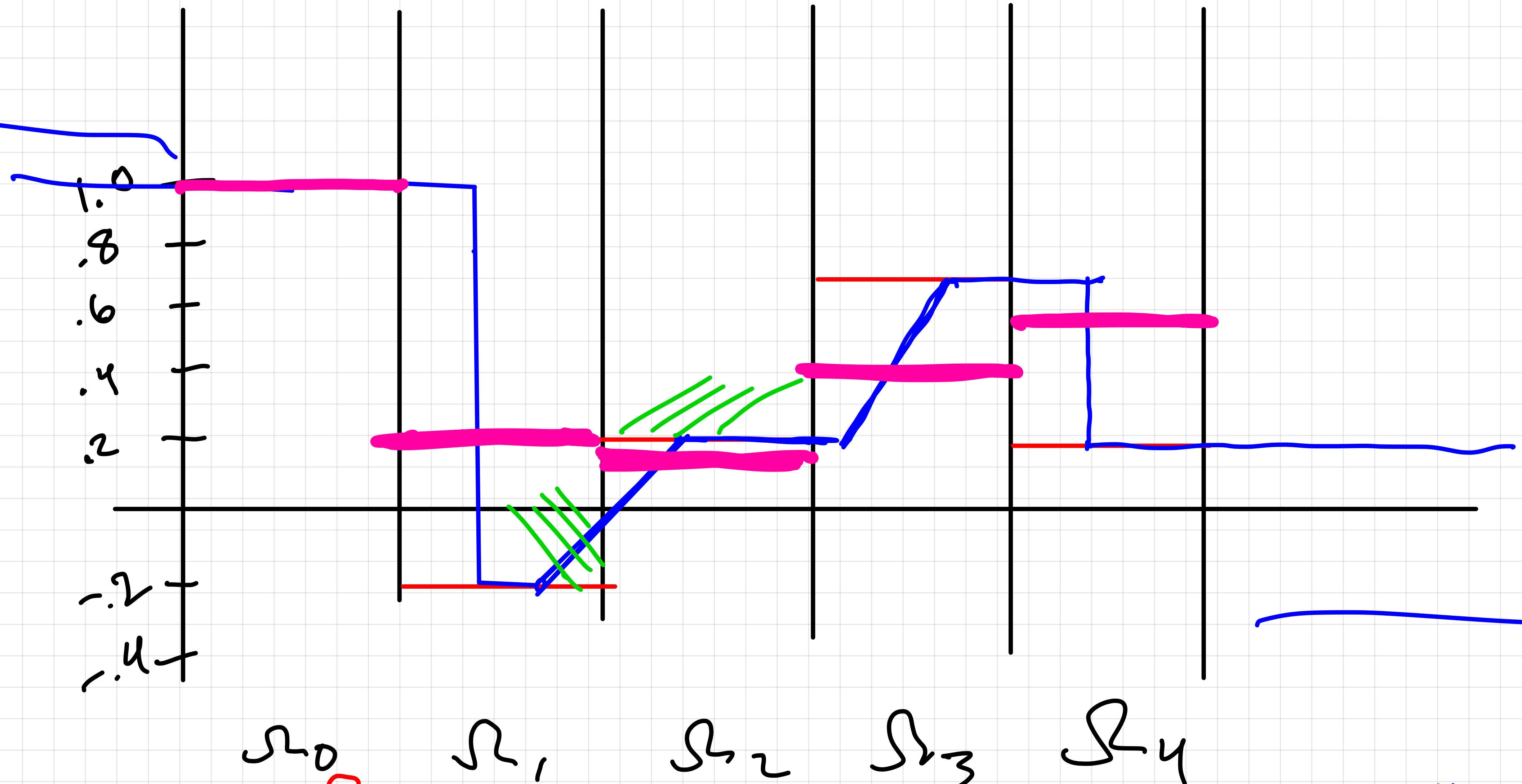
Godunov's Method:

Consider $u_t + \left(\frac{u^2}{2}\right)_x = 0$

$$u(x,0) = \begin{cases} u^- & x \leq 0 \\ u^+ & x > 0 \end{cases}$$

$$\text{if } u^- > u^+ : u(x,t) = \begin{cases} u^- & \text{if } x < st, \\ u^+ & \text{if } x > st, \end{cases}$$

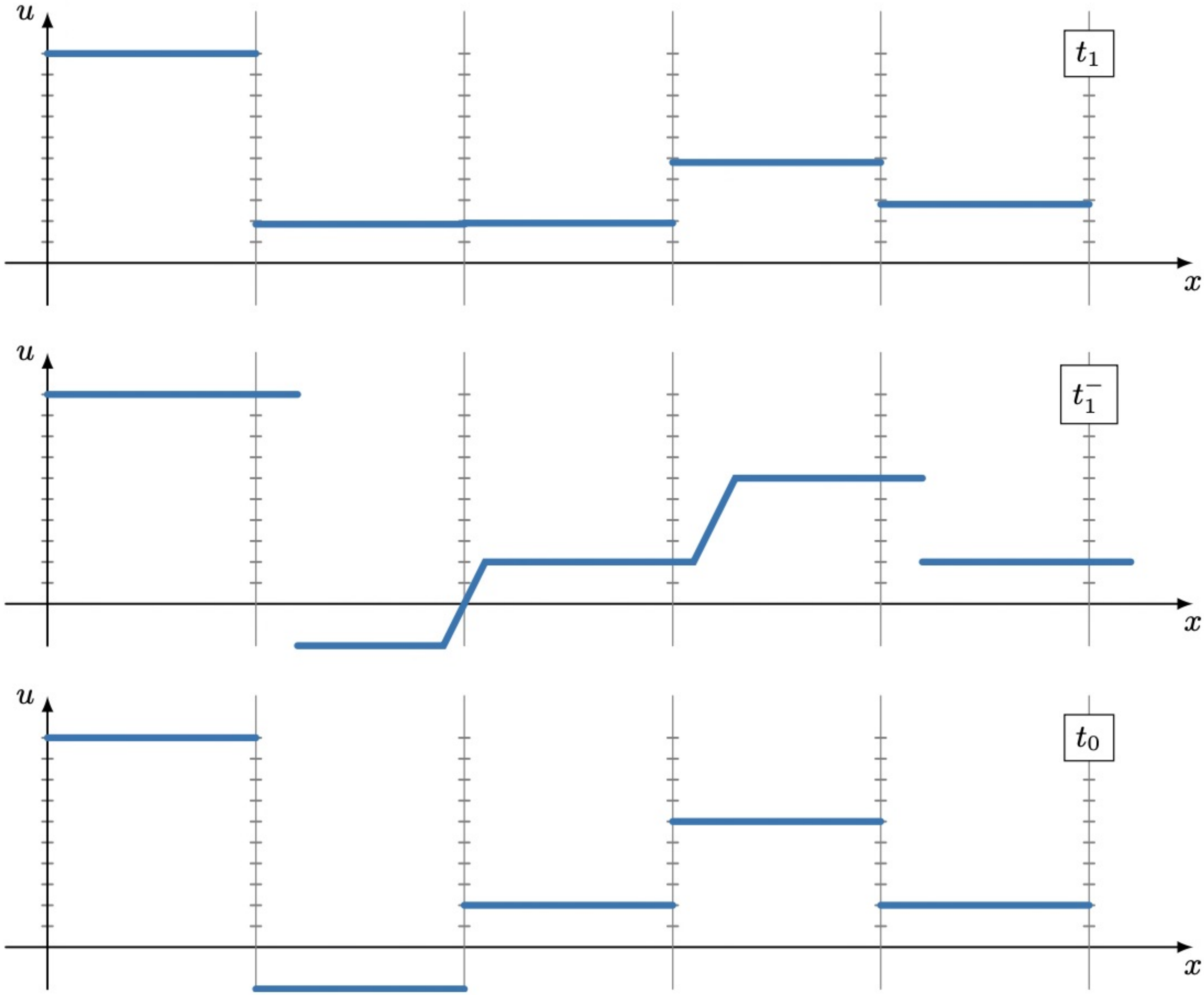
$$\text{if } u^- \leq u^+ : u(x,t) = \begin{cases} u^- & \text{if } x < u^- t, \\ x/t & \text{if } u^- t \leq x \leq u^+ t, \\ u^+ & \text{if } u^+ t < x. \end{cases}$$

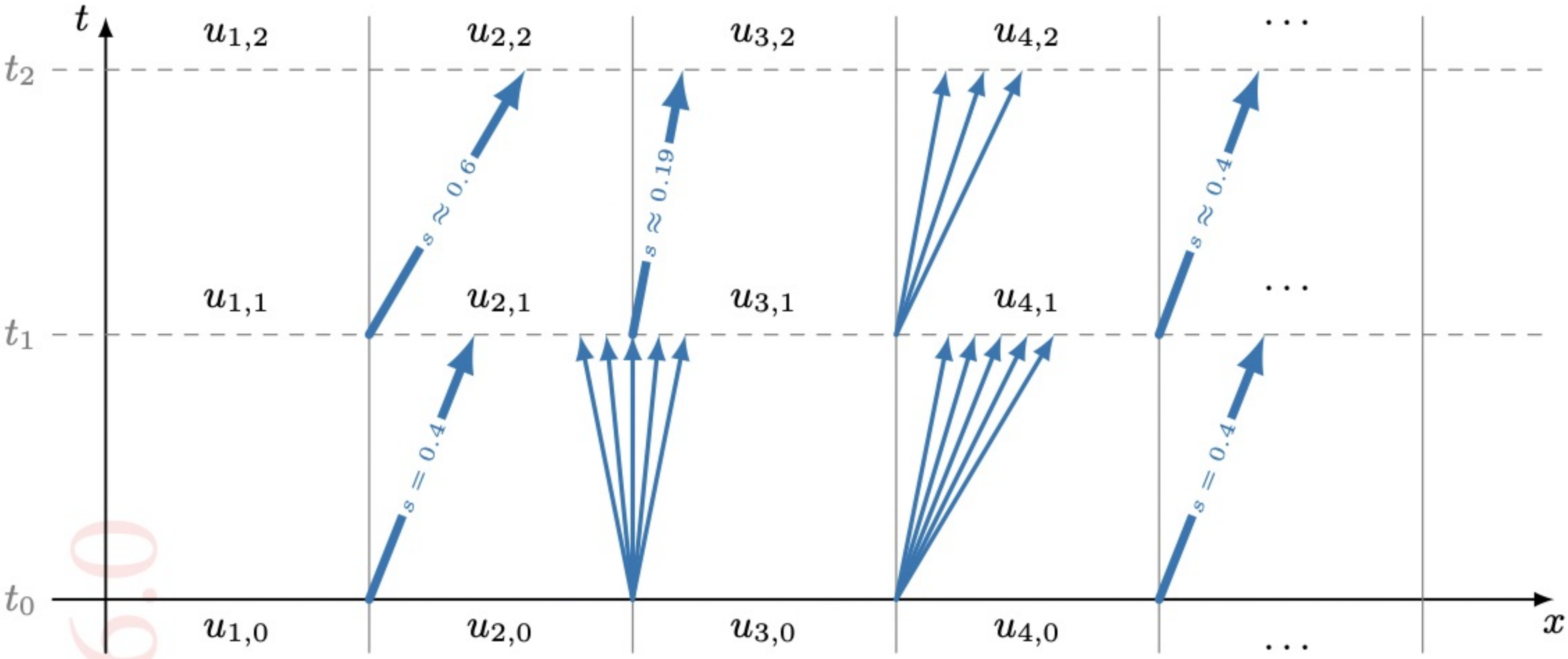


$$u(x,0) = \begin{cases} 1 & x \in \Omega_0 \\ -.2 & x \in \Omega_1 \\ .2 & x \in \Omega_2 \\ .6 & x \in \Omega_3 \\ .2 & x \in \Omega_4 \end{cases}$$

<u>Step</u>

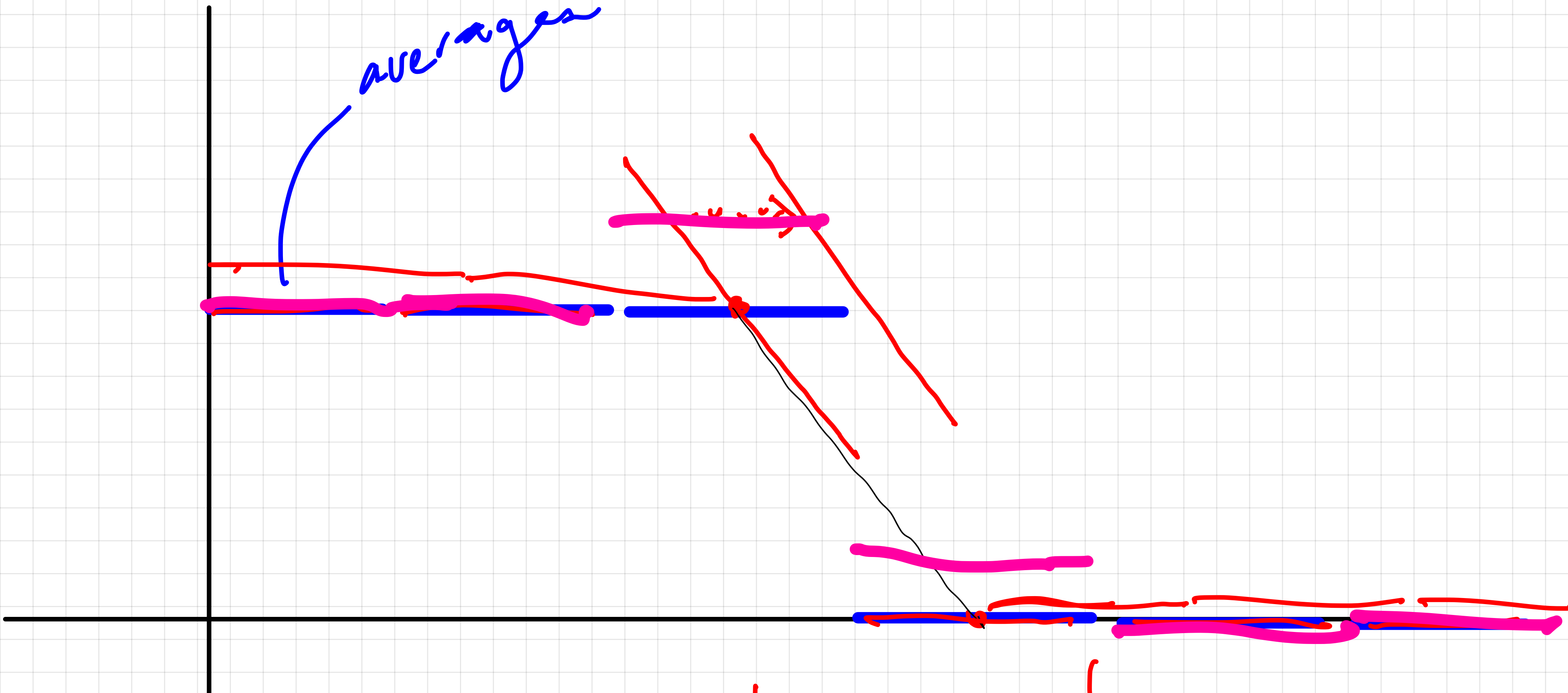
1. draw init condition
2. Evolve all 4 Riemann problems
3. Compute avg per cell


u
t_1
x
u
t_1^-
x
u
t_0
x



t
x
t_0
t_1
t_2
$u_{1,0}$
$u_{2,0}$
$u_{3,0}$
$u_{4,0}$
$u_{1,1}$
$u_{2,1}$
$u_{3,1}$
$u_{4,1}$
$u_{1,2}$
$u_{2,2}$
$u_{3,2}$
$u_{4,2}$
$s = 0.4$
$s \approx 0.6$
$s \approx 0.19$
$s \approx 0.4$
$s = 0.4$

$$u_t + \left(\frac{u^2}{2}\right)_x = 0$$



- Reconstruct solution as <u>linear</u>
- Evolve
- Average