in 1D

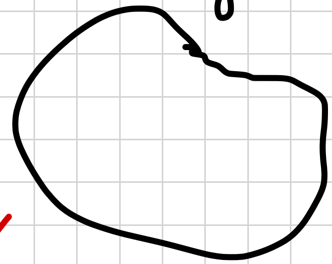
$$-u_{xx} = f \quad \text{on } [0,1]$$
$$u(0) = 0$$
$$u'(1) = 0$$

$$a(u,v) = \int_0^1 u_x v_x \, dx$$
$$\langle f, v \rangle = \int_0^1 f v \, dx$$

Find $u \in V$ s.t.

$$a(u,v) = \langle f, v \rangle \quad \forall v \in V.$$

in 2D/3D: let Ω be open, connected



let $f \in \cancel{L^2}$

$\in H^1(\Omega)$ = space where $f \in L^2$

$f' \in L^2$

$$\begin{cases} -\nabla \cdot \nabla u = f & \text{in } \Omega \\ u(\underline{x}) = 0 & \text{on } \partial\Omega \end{cases}$$

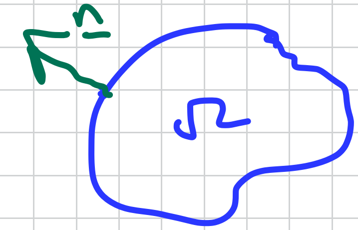
<u>2D</u>

$$-\Delta = -\nabla \cdot \nabla = -\underbrace{\begin{bmatrix} \partial_x & \partial_y \end{bmatrix}}_{\nabla \cdot} \begin{bmatrix} \partial_x \\ \partial_y \end{bmatrix}$$

$$= -\partial_{xx} - \partial_{yy}$$

$$-\nabla\cdot\nabla u = f$$

$$u(\underline{x}) = 0 \quad \text{on } \partial\Omega .$$



$$\int_\Omega -\nabla\cdot\nabla u \; v \; d\underline{x} = \int_\Omega f \, v \, d\underline{x}$$

IBP: $$\underbrace{\int_\Omega \nabla\cdot(a\,\underline{b})}_{\int_\Omega \nabla a\cdot b \,+\, \int_\Omega a\nabla\cdot b} = \int_{\partial\Omega} \underline{n}\cdot(a\,\underline{b})$$

$$\int_\Omega \nabla u\cdot\nabla v \; d\underline{x} - \int_{\partial\Omega} \underline{n}\cdot\nabla u \, v \, ds = \int_\Omega f v \, d\underline{x}$$

impose $v(\underline{x}) = 0$ on $\partial\Omega$

on v

$$\int_\Omega \nabla u\cdot\nabla v \, dx = \int_\Omega f v \, dx$$

Galerkin: find $u \in H^1$ s.t.

$$\int_\Omega \nabla u \cdot \nabla v \, d\underline{x} = \int_\Omega f v \, dx \quad \forall v \in H^1_0$$

Mixed form: $-\nabla \cdot \nabla u = f$

$$\rightarrow \begin{cases} \underline{q} - \nabla u = f \\ -\nabla \cdot \underline{q} = 0 \end{cases}$$

Least-squares: minimize $\| f + \nabla \cdot \nabla u \|_0^2 \rightarrow \min$

or

$\| f - \underline{q} + \nabla u \|_0^2 + \| \nabla \cdot \underline{q} \|_0^2 \Rightarrow \min$