CS556: Iterative Methods: Tentative Schedule, Fall 2022

Week 1–2:

- Course Overview
- Introduction / Examples
  - A first iterative example
  - A second iterative example: 1D Poisson
  - grid-function evaluation as matrix-vector products
  - stencils
  - Kronecker Product Introduction
- The Geometry of Linear Systems
- Saad Chapter 1, Linear Algebra basics
  - norms
  - diagonally dominant matrices, irreducibly diagonally dominant
  - M matrices
  - eigenvalues / eigenvectors / Schur form, etc.
  - projection: 1D / n-D
- Gaussian elimination
  - Full systems
  - Banded systems

Week 2–3:

- Saad Chapter 2: Discretization of PDEs
  - Finite differences
    - Poisson / Helmholtz / Advection-diffusion 1D
    - Poisson 2D
    - Poisson 3D
    - Eigenvalues
  - Finite elements
    - Poisson 2D / 3D
  - Direct methods and the curse of dimensionality
  - Using Kronecker products for fast Poisson solvers
- HPC considerations (Saad Chapter 11.2)
  - pipelined/vectorized arithmetic
  - memory hierarchies (caches)
  - interpretive languages
  - examples that slow performance

Week 4:

- Sparse Matrices
  - Graph representations
  - Sparse-matrix formats (CSR, etc.)
- Reordering
  - RCM, nested dissection

  - Impact of reordering on matrix fill
- Sparse direct methods
  - minimum degree ordering
  - nested dissection ordering
  - A-conjugacy of ND orderings

Week 5: Basic Iterative Methods

- Jacobi, GS, SOR
- Jacobi vs GS: Poisson v. Advection-Diffusion
- Some convergence results
- ADI: Poisson / Helmholtz

Week 6: Projection Methods

Week 6–7: Conjugate Gradient Iteration

- Derivation and convergence rate
- Unpreconditioned variant
- Preconditioned variant (inc. Jacobi PCG example)
- Relationship to orthogonal polynomials
- Relationship to Steepest Descent
- Relationship to Lanczos
- Flexible CG

Week 8: GMRES

- Full GMRES
- Restarted GMRES
- Flexible GMRES
- Left/Right preconditioning
- Alternatives to GMRES (Saad Chapter 7)

Week 9–11: Preconditioning

- Block Jacobi
- Overlapping Schwarz (additive/multiplicative)
- Multilevel Schwarz
- Substructuring
- ILU

Week 11–13: Multigrid

Remark. We will inter-disperse many of the topics throughout the course, with the intent that the principal discussion and exploration of a given topic be covered in the proposed timeline. The overall aim of the course if for students to be familiar with the tools to efficiently solve large linear systems, which requires knowledge over a broad range of solver technology. Choosing the correct solver can often save an order-of-magnitude in computational costs, so coverage beyond iterative methods important.