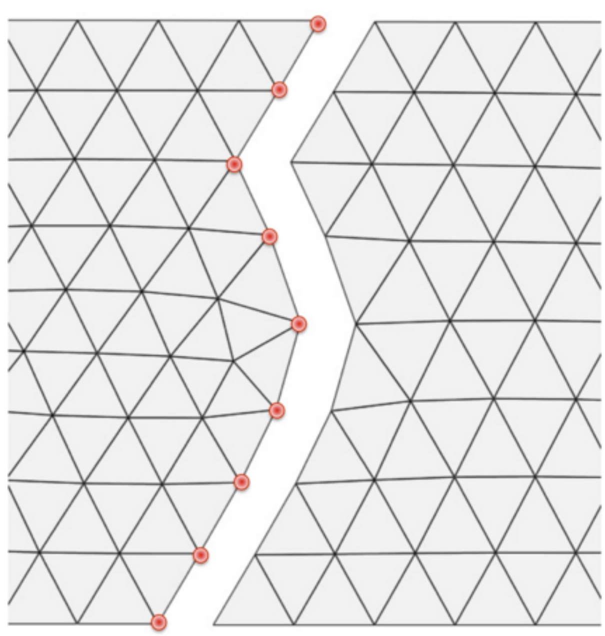
Summary of Notes on Nested Dissection Orderings

- For an $N \times N$ grid in 2D or $N \times N \times N$ grid in 3D, we have the following complexities:

	Lexicographical	Nested-Dissection
2D Factor Cost	$\sim 2n^2$	$\sim (19.07\ldots)n^{\frac{3}{2}}$
2D Solve & Storage Cost	$\sim 2n^{\frac{3}{2}}$	$\sim \frac{31}{8} n \log_2 n$
3D Factor Cost	$\sim 2n^{\frac{7}{3}}$	$O(n^2)$
3D Solve & Storage Cost	$\sim 2n^{\frac{5}{3}}$	$O(n^{\frac{4}{3}})$

  - Note that the constants in the lexicographical case are very small.
  - The constants for the nested dissection are less clear. (Martinsson's notes do not account for the complexity of the side blocks, $L_{ii}^{-1} A_{i\Gamma}$.)
  - The constants for the 2D nested dissection are from A. George's 1973 paper.
  - Note that $20n^{\frac{3}{2}} < 2n^2$ for $n > 100$—meaning it starts to pay for 2D problems that are larger than 10×10.
  - The costs, however, will be relatively (2-10×) higher for nested-dissection because indirect addressing will increase memory traffic and inhibit pipelined arithmetic. So N.D. may require a slightly larger value of N to be competitive. This hypothesis could be tested in matlab.

- The method can be extended to unstructured graphs, e.g., finite-elements.



- The method works for variable-coefficient problems—no change to the algorithm (at least, for the positive-definite case).
- Ordering of the subdomain interior DOFs can influence the fill.
- For the constant coefficient case, fast Poisson solvers are *much* faster, with $O(n \log n)$ complexity for the uniform grid case (`fishpack`), or $\sim 12n^{\frac{4}{3}}$ (all matrix-matrix products) if the grid is based on a 3D tensor product of nonuniform one-dimensional grids.
- Three articles/notes: George'73, Martinsson '14, Schmitz & Ying '14.

The articles are already on the Relate page.