

CS556 Quiz 6

Due 5 PM WEDNESDAY, Oct. 16

1. Consider solution of the 1D heat equation, $-u_{xx} = f(x)$, $u(0) = u(1) = 0$ with E linear finite elements. Answer these two questions for each of the matrices, A_L , \bar{A} , and A .

- What is the dimension of each matrix?
- What is the dimension of the null space?

a. $A_L = \text{block-diag}(A^e)$

b. $\bar{A} = Q^T A_L Q$

c. $A = R \bar{A} R^T$

2. Consider the finite element discretization of the 1D heat equation, $-u_{xx} = f(x)$ with a Dirichlet condition at $x = 0$, $u(0) = 0$, and a Neumann condition at $x = L$, $u_x(L) = 0$. Suppose $L = 10$ and $E = 5$ with uniform element length Δx .

- a. What is the stiffness matrix, A , in this case? Derive this by hand. (It is easier to do so in *global* form, rather than by assembling the local stiffness matrix.)
- b. How many degrees-of-freedom (unknowns) do you have for this problem?
- c. How many degrees-of-freedom do you have when using this same mesh but with Dirichlet boundary conditions at $x = 0$ and L ?