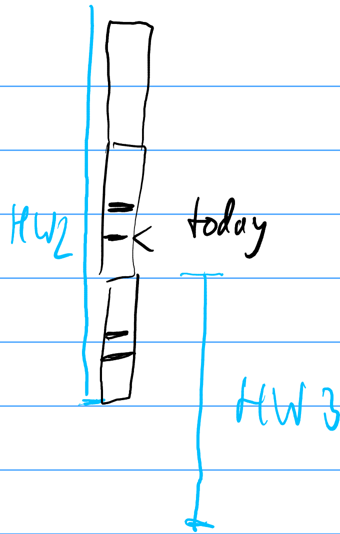
Today

- HW2/3 timeline
- Projects
- Mpole/local
- CA mpole/local
- fast alys

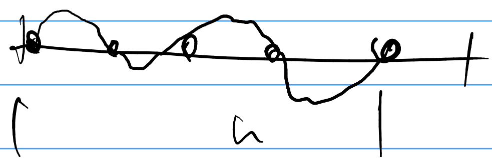


Error in local: $\left(\frac{dft}{dcs}\right)^{k+1}$

update: $\left(\frac{dfs}{dct}\right)^{k+1}$

p-th order accurate method

Error $\leq C \cdot h^p$



Local:

$$\Psi(x-y) = \sum_{|\nu| \leq k} \frac{D_x^\nu \Psi(x-y)|_{x=c}}{\nu!} (x-c)^\nu$$

x: targets

y: sources

$$\Psi(x-y) = \sum_{|\nu| \leq k} \frac{D_y^\nu \Psi(x-y)|_{y=c}}{\nu!} (y-c)^\nu \, (-1)^?$$

Dipole:

$$\lim_{h \to 0} \frac{\Psi(x+h) - \Psi(x-h)}{h}$$

Taylor on Potentials, Again

Stare at that Taylor formula again.

On Rank Estimates

So how many terms do we need for a given precision ε?

Estimated vs Actual Rank

Our rank estimate was off by a power of $\log \varepsilon$. What gives?

Being Clever about Expansions

How could one be clever about expansions? (i.e. give examples)

Making Multipole/Local Expansions using Linear Algebra

Actual expansions seem vastly cheaper than LA approaches. Can this be fixed?

$$u(x_i) = \sum_{j=1}^{S} \underbrace{\psi(x_i - y_j)}_{A_{ij}} \sigma_j$$

$x_1, \dots, x_T$

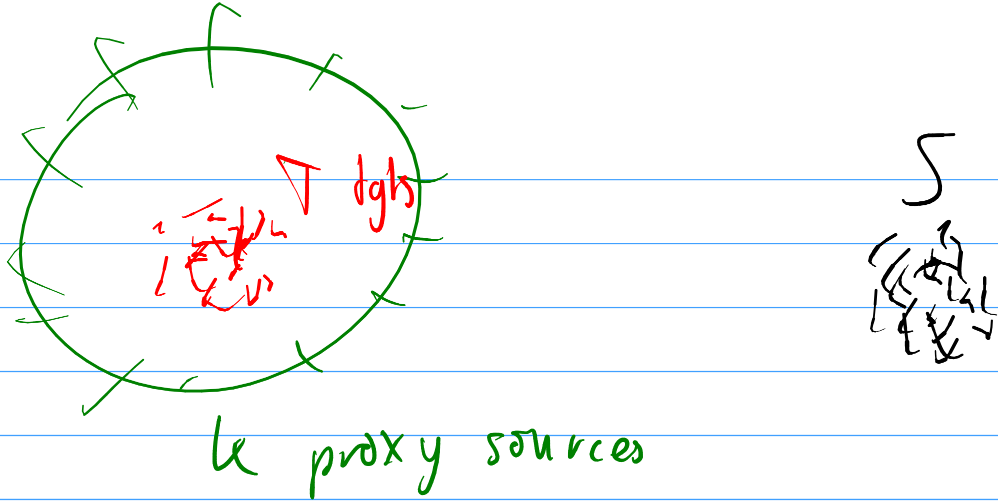
Using LA:

Complexity of analytical exp:

Form: SK

Eval: TK

(K terms)



$$T \;\; \underbrace{\begin{bmatrix} \\ \tilde{A} \\ \\ \end{bmatrix}}_{k} = P\,\hat{\tilde{A}}_{[J,:]}$$

Why Does the Proxy Trick Work?

In particular, how general is this? Does this work for any kernel?

Where are we now?

Summarize what we know about interaction ranks.

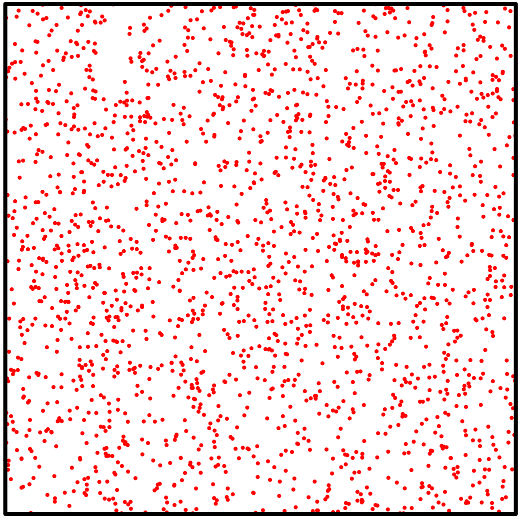
4 Near and Far: Separating out High-Rank Interactions

Simple and Periodic: Ewald Summation

Want to evaluate potential from an infinite periodic grid of sources:

$$\psi(\mathbf{x}) = \sum_{\mathbf{i}\in\mathbb{Z}^d} \sum_{j=1}^{N_{\text{src}}} G(\mathbf{x}, \mathbf{y}_j + \mathbf{i})\varphi(\mathbf{y}_j)$$

Barnes-Hut: Putting Multipole Expansions to Work



(Figure credit: G. Martinsson, Boulder)

Want: All-pairs interaction.
Caution: In these (stolen) figures: targets sources.
Here: targets and sources.