Today

- skeletonization
  - ↳ local
  - ↳ mpole
- Ewald summation
- Barnes-Hut
- Fast Multipole

▶ HW3 out

▶ HW2 rank finder:
try to design incremental alg.

$$A\Omega_1 \rightsquigarrow Q_1$$

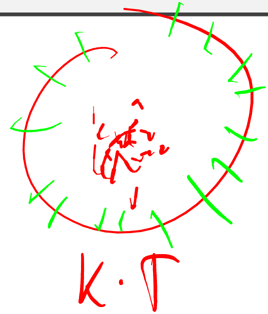
$$(A - Q_1 Q_1^* A)\,\Omega_2 \rightsquigarrow Q_2$$

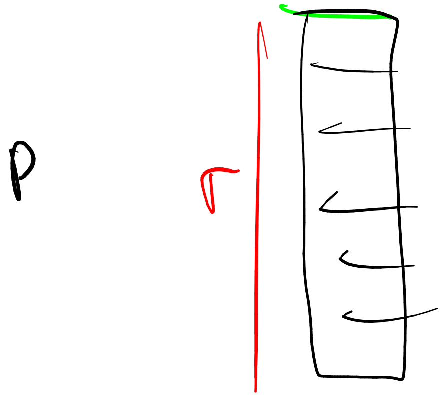
$$A \approx Q_1 Q_1^* A + Q_2 Q_2^* A$$

Making Multipole/Local Expansions using Linear Algebra

Actual expansions seem vastly cheaper than LA approaches. Can this be fixed?

Local exp. via Proxies:





P Interactionmat($T_{[:]}$, S) $\vec{\sigma}$

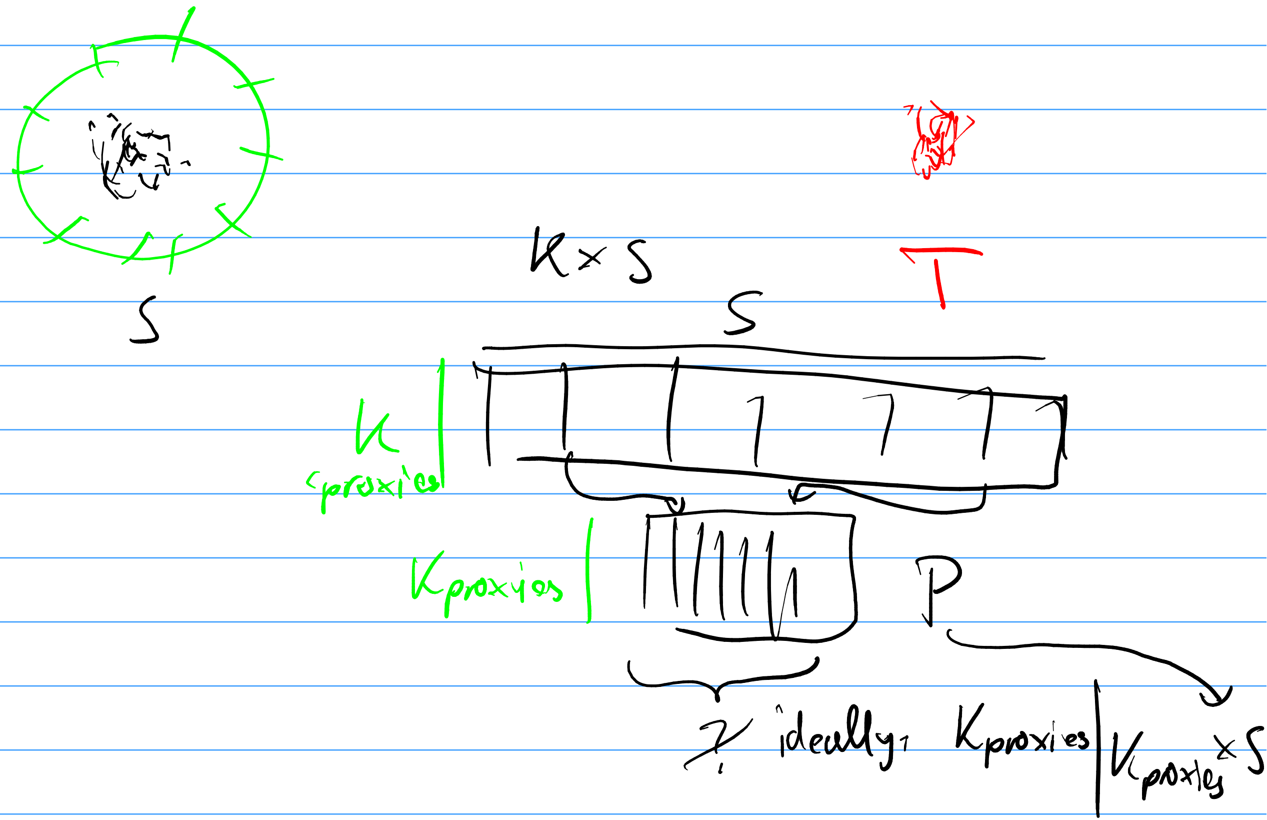
T×k   k×S   S

(T×S)

(k)

( ]

Multipole via proxies:



Why Does the Proxy Trick Work?

In particular, how general is this? Does this work for any kernel?

Where are we now?

Summarize what we know about interaction ranks.

4 Near and Far: Separating out High-Rank Interactions

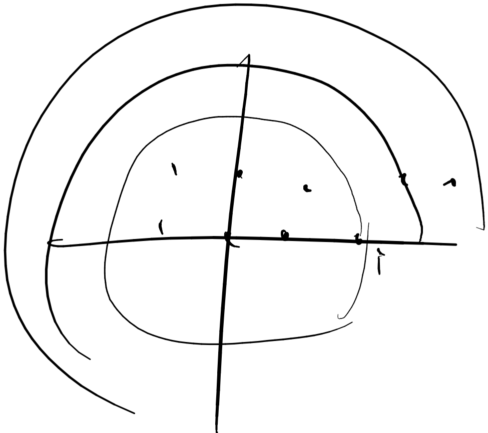
Simple and Periodic: Ewald Summation

Want to evaluate potential from an infinite periodic grid of sources:

$$\psi(\mathbf{x}) = \sum_{\mathbf{i}\in\mathbb{Z}^d} \sum_{j=1}^{N_{\text{src}}} G(\mathbf{x}, \mathbf{y}_j + \mathbf{i})\varphi(\mathbf{y}_j)$$

$\mathbb{Z} \times \mathbb{Z}$

$G(x, y) \approx \frac{1}{|x-y|^p}$

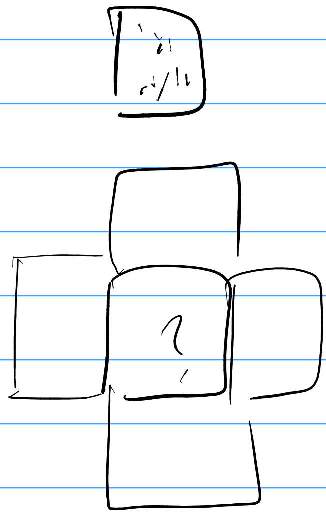


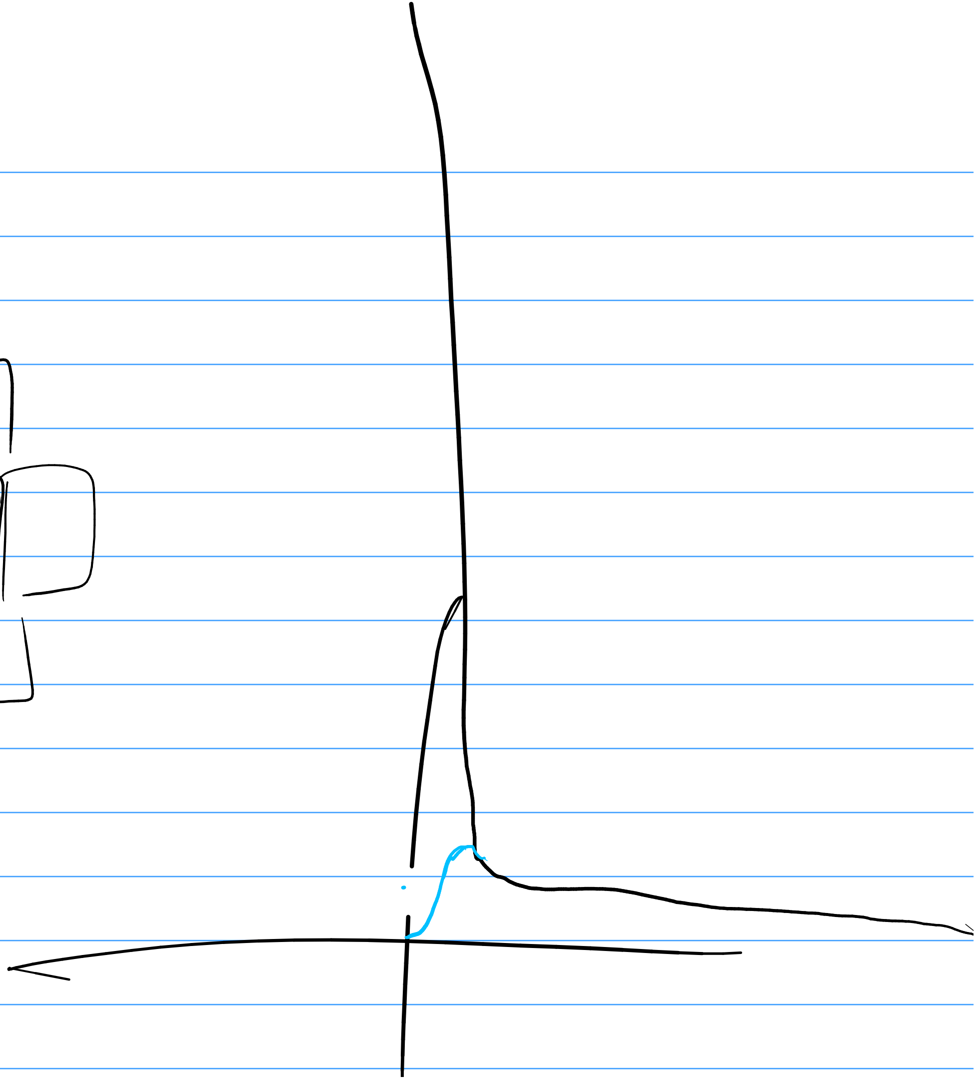
$d-1-p < -1$

$d < p$

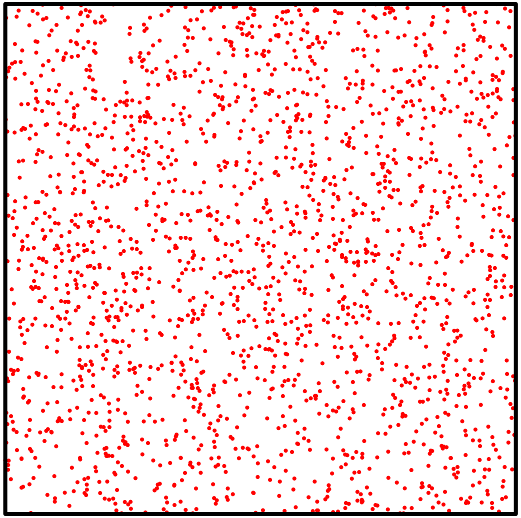
$\sum^{\infty}_{\text{dist on grid}=i,\ i=0} G(x, y_j + i)\, i^{d-1} = \sum_i i^{d-1-p}$, with $G(x, y_j + i) \le \frac{1}{i^p}$

$\sum \frac{1}{n}$





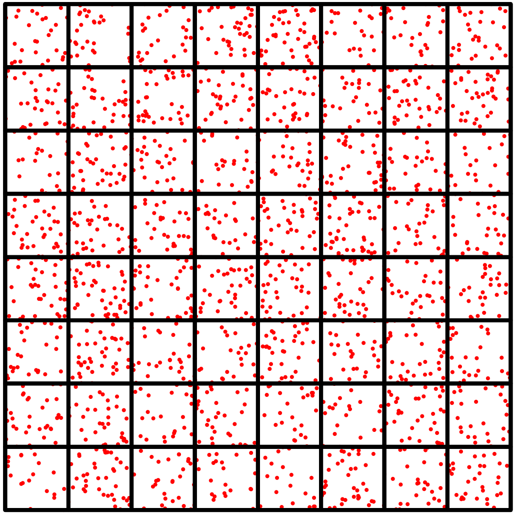
Barnes-Hut: Putting Multipole Expansions to Work



(Figure credit: G. Martinsson, Boulder)

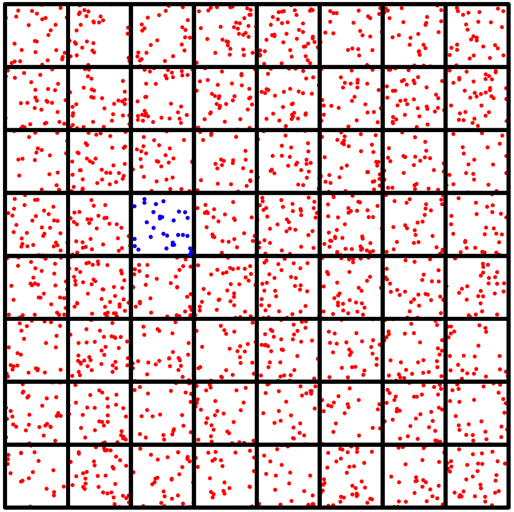
Want: All-pairs interaction.
Caution: In these (stolen) figures: targets sources.
Here: targets and sources.

Barnes-Hut: Putting Multipole Expansions to Work



(Figure credit: G. Martinsson, Boulder)

Barnes-Hut: Putting Multipole Expansions to Work

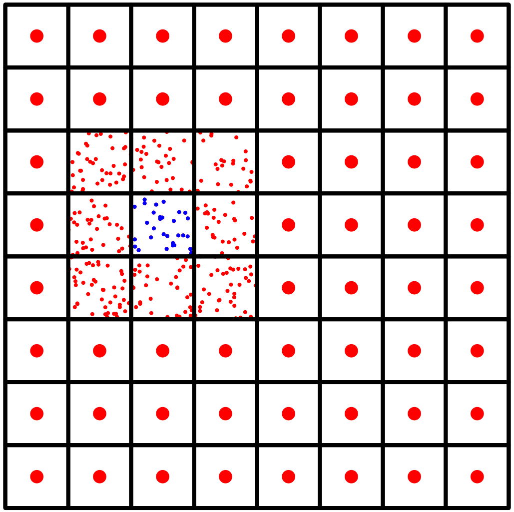


(Figure credit: G. Martinsson, Boulder)

For sake of discussion, choose one ‘box’ as targets.

Q: For which boxes can we then use multipole expansions?

Barnes-Hut: Putting Multipole Expansions to Work



(Figure credit: G. Martinsson, Boulder)

With this computational outline, what's the accuracy?