Today

FMM

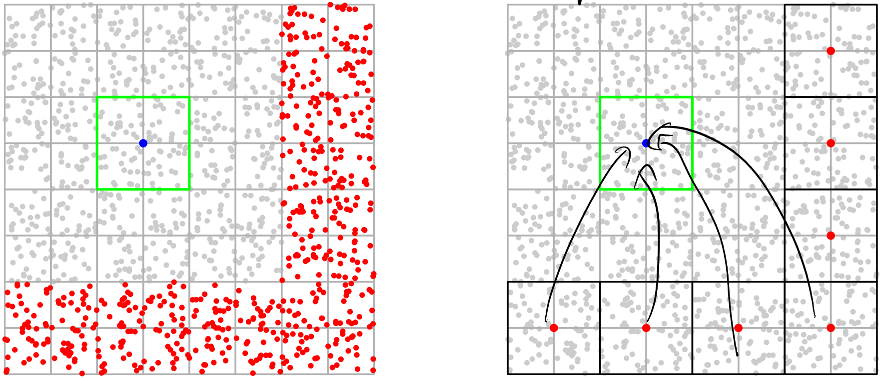
Fast solve

Announcements

Project Proposals!

Using Multipole-to-Local

"Interaction List"
Multipole-to-local: "List 2"



(Figure credit: G. Martinsson)

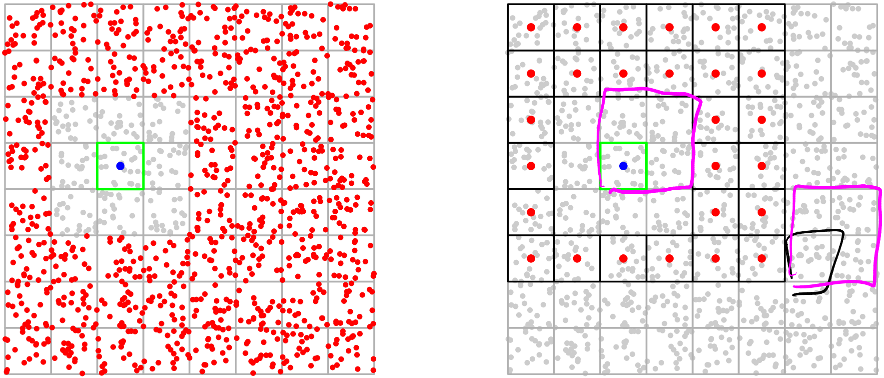
Come up with an algorithm that computes the interaction in the figure.

Using Multipole-to-Local

Come up with an algorithm that computes the interaction in the figure.

1. Compute multipoles and propagate upward
2. Evaluate M2L translations for List 2
   List V
3. Propagate downward
4. Neighbor boxes via P2P ("List 1")
   List U

Using Multipole-to-Local: Next Level



(Figure credit: G. Martinsson)

Assuming we retain information from the previous level, how can we obtain a valid local expansion on the target box?

Using Multipole-to-Local: Next Level

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Define 'Interaction List'

List 2

For a box b, the interaction list I_b consists of all boxes b' so that

- b and b' are on the same level
- b and b' are well-separated (w-s is symmetric!)
- the parents of b and b' touch

The Fast Multipole Method ('FMM')

Upward pass

1. Build tree (Whoops! $O(n \log n)$)
2. Compute interaction lists
3. Compute lowest-level multipoles from sources
4. Loop over levels $\ell = L-1, \dots, 2$:
   4.1 Compute multipoles at level ℓ by mp $\to$ mp

Downward pass

1. Loop over levels $\ell = 2, 3, \dots, L-1$:
   1.1 Loop over boxes b on level ℓ :
      1.1.1 Add contrib from I_b to local expansion by mp $\to$ loc
      1.1.2 Add contrib from parent to local exp by loc $\to$ loc
2. Evaluate local expansion and direct contrib from 9 neighbors.

Overall algorithm: Now $O(N)$ complexity.

Note: L levels, numbered $0, \dots, L-1$. Loop indices above *inclusive*.

What about adaptivity?

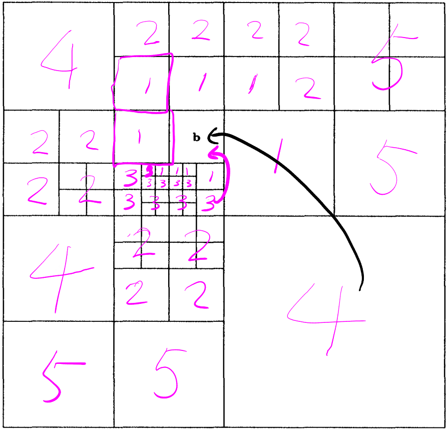


Figure credit: Carrier et al. ('88)

List 1: direct
List 2: M2L
List 3: M2P
List 4: P2L

What about adaptivity?

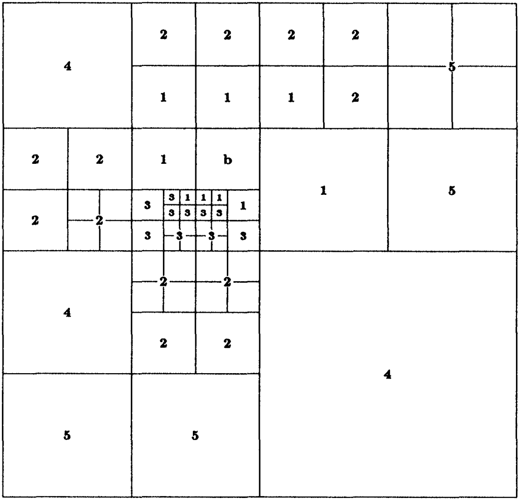
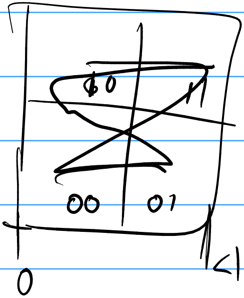


Figure credit: Carrier et al. ('88)

Adaptivity: what changes?

- Two special cases:
  source too big (Lift 4)
  target too big (Lift 3)

Z-order



$0.\underbrace{a\ c\ e\quad\quad}$ $\cdot 2^{0}$

$0.\underbrace{b\ d\ f\quad\quad}$

$0.abcdef$

FMM: List of Interaction Lists

Make a list of cases:

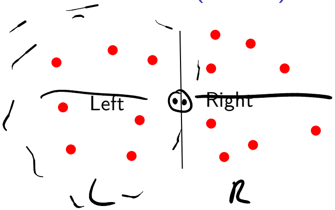
What about solving?

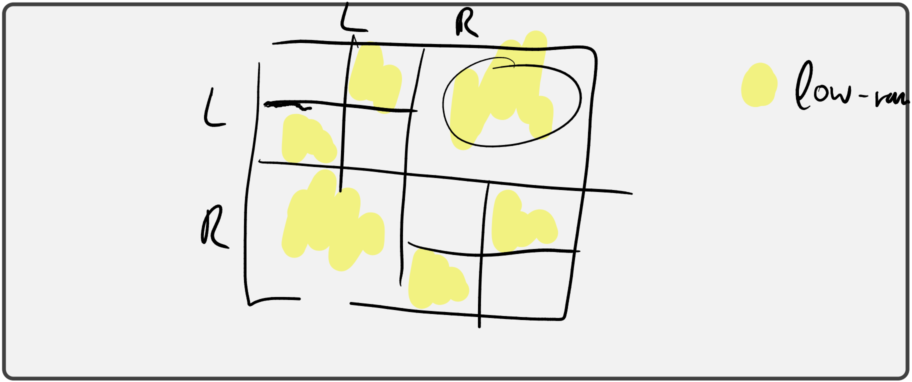
Likely computational goal: Solve a linear system $Ax = b$. How do our methods help with that?

A Matrix View of Low-Rank Interaction

Only *parts of the matrix are low-rank*! What does this look like from a matrix perspective?

(Recursive) Coordinate Bisection (RCB)





Block-separable matrices

$$A = \begin{bmatrix} D_1 & A_{12} & A_{13} & A_{14} \\ A_{21} & D_2 & A_{23} & A_{24} \\ A_{31} & A_{32} & D_3 & A_{34} \\ A_{41} & A_{42} & A_{43} & D_4 \end{bmatrix}$$

where A_{ij} has low rank: How to capture rank structure?

$$A_{ij} \approx (A_{ij})_{(:,J)} \Pi_j$$

Proxy Recap

Saw: If A comes from a kernel for which Green's formula holds, then the same skeleton will work for all of space, for a given set of sources/targets. What would the resulting matrix look like?