

# Announcements:

HW2

## Goals:

- Do expn. stiff via NLA
- Fast alg:
  - Euclid
  - Barnes-Hut

## Review: S

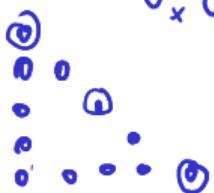
$$\psi(x) = \sum G(x, y) \phi_j$$

$\uparrow O(\kappa)$        $\uparrow O(T\kappa)$



$$\Delta u = 0$$
$$\partial_x^2 u = -\partial_y^2 u$$

$$\Delta \partial_x^{\alpha} \partial_y^{\beta} u = \partial_x^{\alpha} \partial_y^{\beta} \Delta u = 0$$
$$\partial_x^{\gamma} \partial_y^{\delta} u = -\partial_x^{\delta} \partial_y^{\gamma} u$$



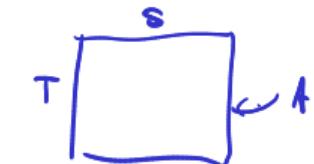
$\begin{pmatrix} \vdots & \vdots \\ \vdots & \vdots \end{pmatrix}$

S

$\begin{pmatrix} \vdots & \vdots & \vdots \\ \vdots & \text{red} & \vdots \\ \vdots & \text{green} & \vdots \\ \vdots & \text{blue} & \vdots \\ \vdots & \text{purple} & \vdots \\ \vdots & \text{orange} & \vdots \\ \vdots & \text{yellow} & \vdots \\ \vdots & \text{pink} & \vdots \\ \vdots & \text{cyan} & \vdots \\ \vdots & \text{magenta} & \vdots \\ \vdots & \text{black} & \vdots \\ \vdots & \text{brown} & \vdots \\ \vdots & \text{gray} & \vdots \\ \vdots & \text{teal} & \vdots \\ \vdots & \text{light blue} & \vdots \\ \vdots & \text{dark blue} & \vdots \\ \vdots & \text{purple} & \vdots \\ \vdots & \text{pink} & \vdots \\ \vdots & \text{cyan} & \vdots \\ \vdots & \text{magenta} & \vdots \\ \vdots & \text{black} & \vdots \\ \vdots & \text{brown} & \vdots \\ \vdots & \text{gray} & \vdots \\ \vdots & \text{teal} & \vdots \\ \vdots & \text{light blue} & \vdots \\ \vdots & \text{dark blue} & \vdots \end{pmatrix}$

Form the  $T \times S$  matrix

Want: row / target subset



Idea: Use proxies to find target subset

$$P A_{\text{tgt}} \approx A$$

Proxies

A diagram of a matrix with T rows and S columns. The rows are labeled with T at the top and S at the bottom right. Only one specific row is highlighted with a thick pink line. A curved arrow points from the word "proxies" to this highlighted row.

$$\psi(x) = G(x_i, y_j) \sigma_j^T$$

$A \approx$

$$\approx A_{[i,j]} P \approx$$

$A \approx A_{[i,j]} P$

# The Proxy Trick

Idea: *Skeletonization using Proxies*

Demo: *Skeletonization using Proxies*

Q: What error do we expect from the proxy-based multipole/local 'expansions'?

Use  $H$  proxies terms in "good" Taylor,  
fine accuracy

# Why Does the Proxy Trick Work?

In particular, how general is this? Does this work for any kernel?

Non-PDE kernels?

# proxies scales with  $O(p^{d-1})$ ,  
not  $O(p^d)$   
... so a PDE is needed.

What happened?



potential  
represented by  
nearby  
equivalent sources

Why O(1)?

Green's formula



If  $\Delta u = 0$  on  $\partial\Omega$ , then

$$(x \in \Omega) \quad u(x) = \pm \int_{\partial\Omega} G(x, y) \frac{\partial u(y)}{\partial n} dS_y, \quad \text{at } x$$
$$+ \int_{\partial\Omega} \frac{\partial G(x, y)}{\partial n_y} u(y) dS_y$$

↑ ↑

$dS_y$   $D(\frac{\partial u}{\partial n})$

Message: explicitly constructed  
equivalent sources

## Where are we now? (I)

Summarize what we know about interaction ranks.

- ▶ We know that far interactions with a smooth kernel have low rank.  
(Because: short Taylor expansion suffices)
- ▶ If

$$\psi(\mathbf{x}) = \sum_j G(\mathbf{x}, \mathbf{y}_j) \varphi(\mathbf{y}_j)$$

satisfies a PDE (e.g. Laplace), i.e. if  $G(\mathbf{x}, \mathbf{y}_j)$  satisfies a PDE, then that low rank is even lower.

- ▶ Can construct interior ('local') and exterior ('multipole') expansions (using Taylor or other tools).
- ▶ Can lower the number of terms using the PDE.
- ▶ Can construct LinAlg-workalikes for interior ('local') and exterior ('multipole') expansions.
- ▶ Can make those cheap using proxy points.