

Ann:

- HW3
- Project propose

Review:

- fast matrices
- fast solve



Goals:

- Generic / LA
FFT
- Broad outline:
Integral operators
for PDEs

Recap: Fast Fourier Transform

The *Discrete Fourier Transform (DFT)* is given by:

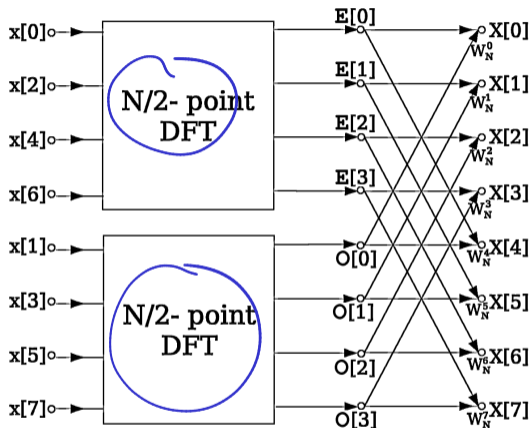
$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} nk} \quad (k = 0, \dots, N-1)$$

The foundation of the *Fast Fourier Transform (FFT)* is the factorization:

$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of even-indexed part of } x_n} + e^{-\frac{2\pi i}{N} k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of odd-indexed part of } x_n} .$$

↑
twiddle factors

FFT: Data Flow

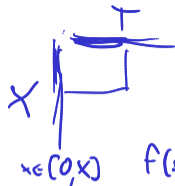
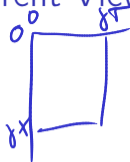


$O(N)$

(Figure credit: [Wikipedia](#))

Perhaps a little bit like a butterfly?

Fourier Transforms: A Different View



$$f(x) = \int_0^T e^{ixt} \varphi(t) dt$$

Claim:

The [numerical] rank of the normalized Fourier transform with kernel e^{ixt} is bounded by a constant times γ , at any fixed precision ϵ .

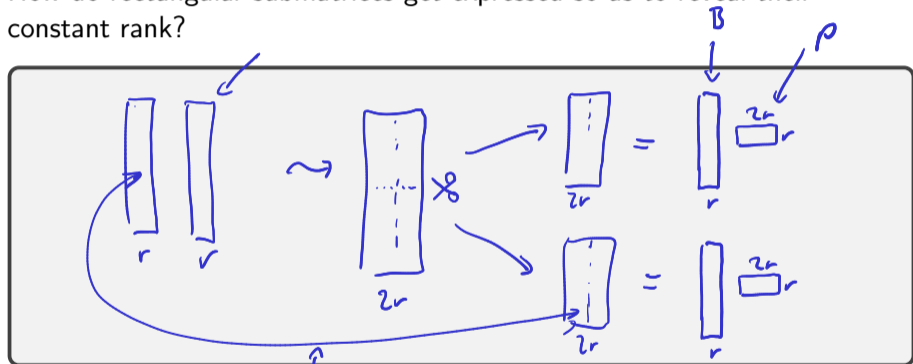
(i.e. rank is proportional to the area of the rectangle swept out by x and t)

[[O'Neil et al. '10](#)]

Demo: Butterfly Factorization (Part I)

Recompression: Making use of Area-Bounded Rank

How do rectangular submatrices get expressed so as to reveal their constant rank?



"same area" \approx # rows \times # cols

Observations

Demo: Butterfly Factorization (Part II)

For which types of matrices is the Butterfly factorization guaranteed accurate?

For which types of $n \times n$ matrices does the butterfly lead to a reduction in cost?

Cost

What is the cost (in the reduced-cost case) of the matvec?

level $l \dots n$:

$$\# \text{ blocks} = 2^L = O(N)$$

$$\begin{aligned} \text{work at this level: } \# \text{ blocks} \cdot \text{block size} \\ = \# \text{ blocks} \cdot (r \times 2r) \end{aligned}$$

$$r = O(1)$$

Comments?

Outline

Introduction

Dense Matrices and Computation

Tools for Low-Rank Linear Algebra

Rank and Smoothness

Near and Far: Separating out High-Rank Interactions

Outlook: Building a Fast PDE Solver

Going Infinite: Integral Operators and Functional Analysis

Singular Integrals and Potential Theory

Boundary Value Problems

Back from Infinity: Discretization

Computing Integrals: Approaches to Quadrature

Going General: More PDEs