Ann:

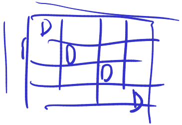
- HW3
- Project proposal

Goals:

- Generic / LA
  FFT
- Broad outline:
  Integral operators
  for PDEs

Review:

- fast matrices
- fast solve



Recap: Fast Fourier Transform

The *Discrete Fourier Transform (DFT)* is given by:

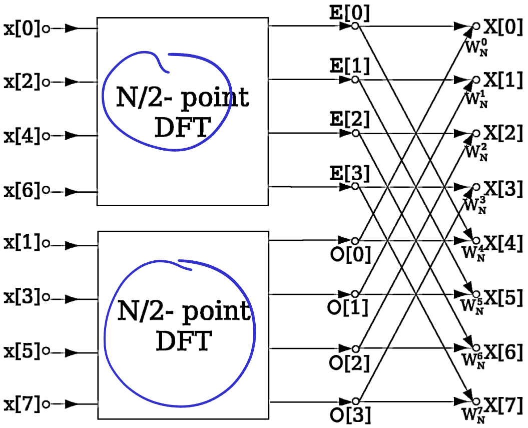
$$X_k = \sum_{n=0}^{N-1} x_n e^{-\frac{2\pi i}{N} nk} \quad (k = 0, \ldots, N-1)$$

The foundation of the *Fast Fourier Transform (FFT)* is the factorization:

$$X_k = \underbrace{\sum_{m=0}^{N/2-1} x_{2m} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of even-indexed part of } x_n} + e^{-\frac{2\pi i}{N} k} \underbrace{\sum_{m=0}^{N/2-1} x_{2m+1} e^{-\frac{2\pi i}{N/2} mk}}_{\text{DFT of odd-indexed part of } x_n}.$$

twiddle factors

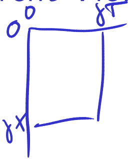
FFT: Data Flow



O(N)

(Figure credit: Wikipedia)
Perhaps a little bit like a butterfly?

Fourier Transforms: A Different View



$x \in [0, X]$ $\quad f(x) = \int_0^T e^{ixt} \varphi(t)\, dt$

Claim:

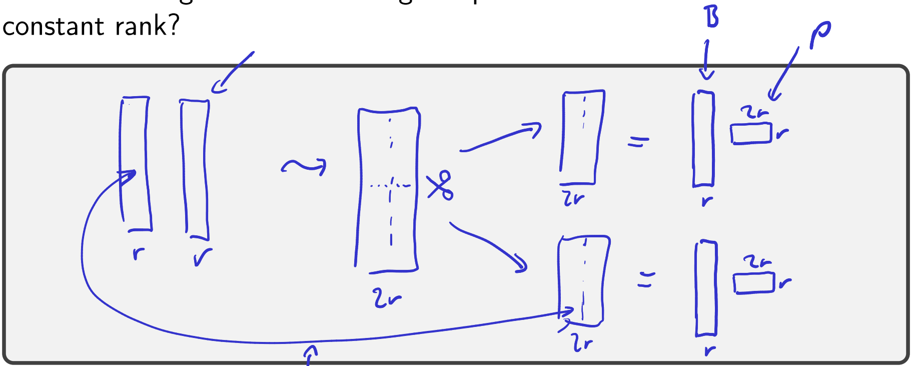
> *The [numerical] rank of the normalized Fourier transform with kernel $e^{i\gamma xt}$ is bounded by a constant times γ, at any fixed precision ϵ.*

(i.e. rank is proportional to the area of the rectangle swept out by x and t)
[O'Neil et al. '10]

Demo: Butterfly Factorization (Part I)

Recompression: Making use of Area-Bounded Rank

How do rectangular submatrices get expressed so as to reveal their constant rank?



"same area" ≈ # rows × # cols

Observations

Demo: Butterfly Factorization (Part II)

For which types of matrices is the Butterfly factorization guaranteed accurate?

For which types of $n \times n$ matrices does the butterfly lead to a reduction in cost?

Cost

What is the cost (in the reduced-cost case) of the matvec?

$r = O(1)$

Level $1 \dots n$:

$\#\text{ blocks} = 2^l = O(N)$

Work at this level: $\#\text{blocks} \cdot \text{block size}$

$= \#\text{blocks} \cdot (r \times 2r)$

Comments?

Outline