Ann:

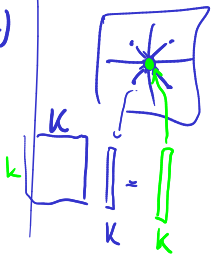
- Hw2 deadline ext. Fri @ noon
- HW3

Goals:

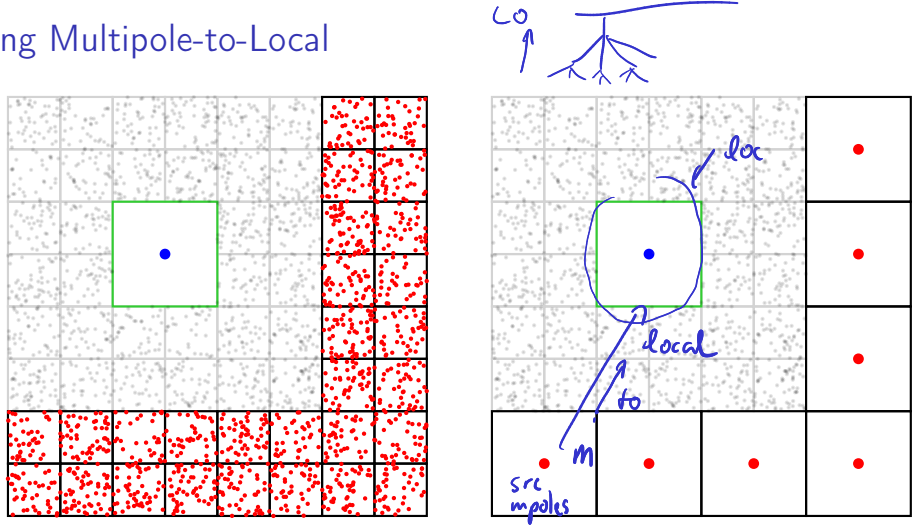
- Fmm (m2l, c2l)
- Solve
  - Iterative solves
  - Direct?

Review:

- Barnes-Hut
  - $O(N \log N)$
  - Stages:
    - compute mpoles
      - V1: start from particles
        - $O(N \log N)$
      - V2:
        - only form for leaves
        - use m2m
        - $O(N)$
    - eval mpoles
    - eval direct interactions from neighbors



Using Multipole-to-Local



(Figure following G. Martinsson)

Come up with an algorithm that computes the interaction in the figure.

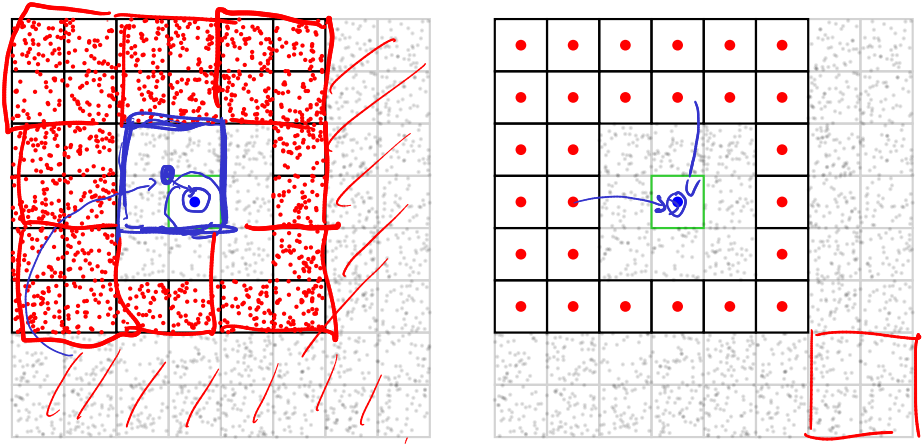
Using Multipole-to-Local

Come up with an algorithm that computes the interaction in the figure.

- Form multipoles
- translate mpole to local
- ? eval local directly
  - ↳ don't want

Instead: reuse already computed local

Using Multipole-to-Local: Next Level



(Figure following G. Martinsson)

Assuming we retain information from the previous level, how can we obtain a valid local expansion on the target box?

Using Multipole-to-Local: Next Level

Assuming we retain information from the previous level, how can we obtain a valid local expansion on the target box?

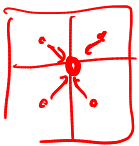
- Obtain mpole contrib for boxes well-sep from parent: loc → loc from parent
- Obtain mpole contributions for newly well-sep boxes (i.e. not well-sep from parent) via mpole → loc.

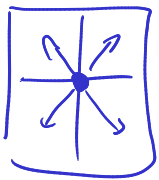
- Keep recursing until leaf,
- At leaf level, pick up near neighbors

Define 'Interaction List'

For a box b, the interaction list I_b consists of all boxes b' so that

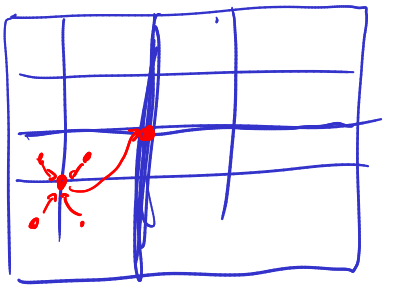
- b' and b are well-separated.
- parent(b') and parent(b) touch
- b' and b are on the same level





Size of interaction list:

$\leq 27. = O(1)$



The Fast Multipole Method ('FMM')

Upward pass

1. Build tree
2. Compute interaction lists
3. Compute lowest-level multipoles from sources
4. Loop over levels $\ell = L-1, \ldots, 2$:
   - 4.1 Compute multipoles at level ℓ by mp $\to$ mp

Downward pass

1. Loop over levels $\ell = 2, 3, \ldots, L-1$:
   - 1.1 Loop over boxes b on level ℓ :
     - 1.1.1 Add contrib from I_b to local expansion by mp $\to$ loc
     - 1.1.2 Add contrib from parent to local exp by loc $\to$ loc
2. Evaluate local expansion and direct contrib from 9 neighbors.

Overall algorithm: Now $O(N)$ complexity.

Note: L levels, numbered $0, \ldots, L-1$. Loop indices above *inclusive*.

What about adaptivity?

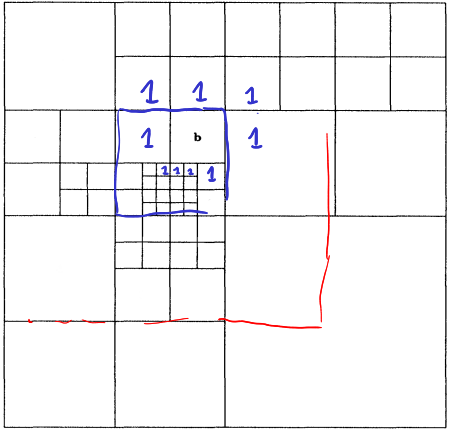


Figure credit: Carrier et al. ('88)

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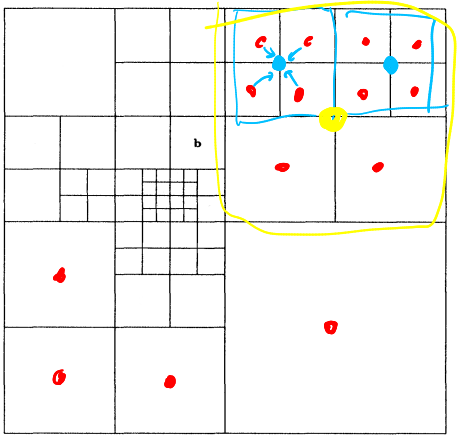

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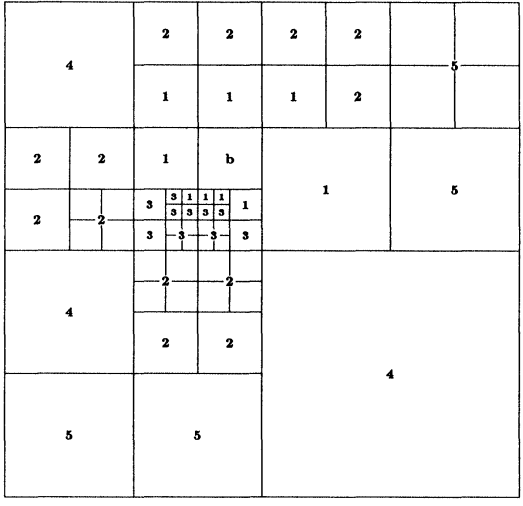

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